



# Relaxation, Band Filling, and Screening in the Transient Dielectric Function of Ge Determined with Femtosecond Ellipsometry

Stefan Zollner

in collaboration with:

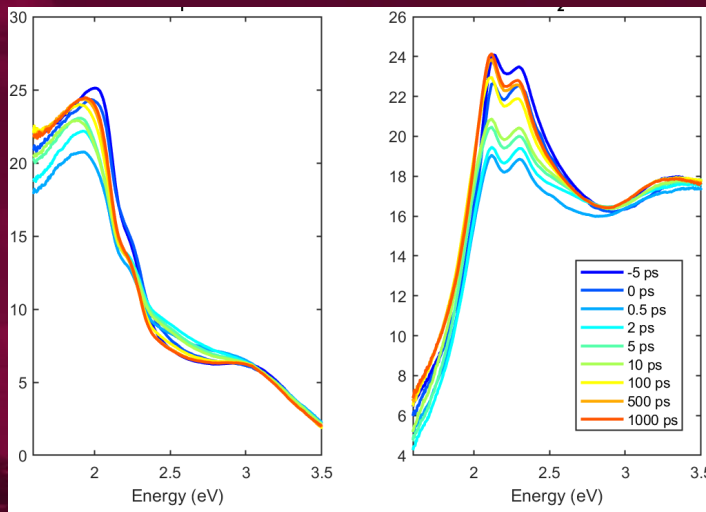
Carola Emminger, Carlos A. Armenta (NMSU)

Shirly Espinoza, Mateusz Rebarz, Martin Zahradnik,

Saul Vazquez-Miranda, Jakob Andreasson (ELI Beamlines)

Steffen Richter, Oliver Herrfurth, Rüdiger Schmidt-Grund

(Leipzig, Ilmenau)

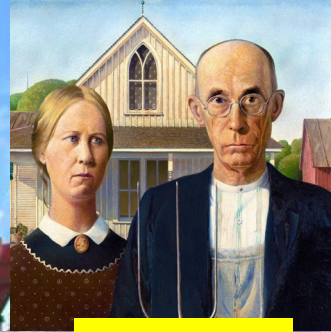


# Biography

Regensburg/Stuttgart  
Germany



NMSU  
Las Cruces, NM  
Since 2010



Ames, IA



Freescale, IBM  
New York, 91-92; 07-10

Motorola, Freescale  
Texas, 2005-2007

Motorola (Mesa, Tempe)  
Arizona, 1997-2005





# Where is Las Cruces, NM ???



When I was a kid, I thought that **if I could find a way to combine physics with New Mexico, my life would be perfect.**

My two great loves are physics and New Mexico. It's a pity they can't be combined.

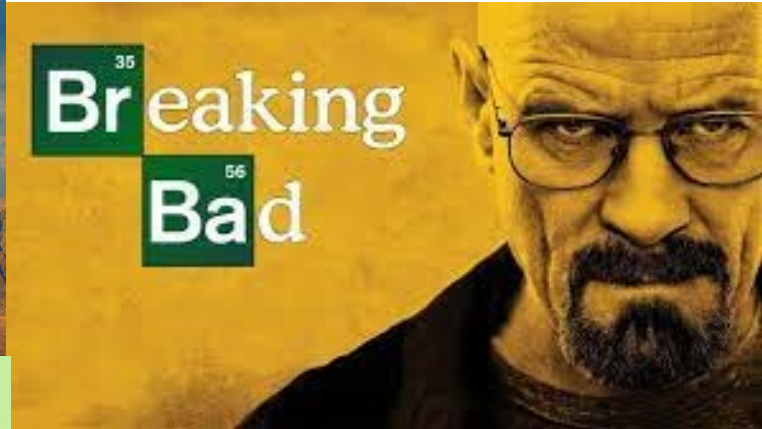
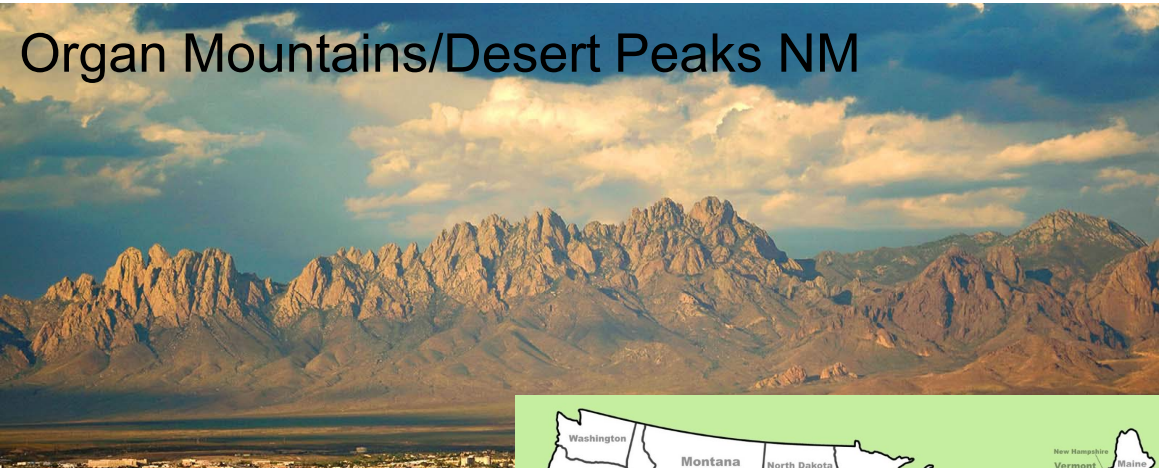
J. Robert Oppenheimer

Note: The NMSU Department of Physics was founded in 1934 by Prof. George Gardiner.

NP



# Where is Las Cruces, NM ???



White Sands NP



Ristra



# Red or green chile peppers?

Green chile  
Capsicum annuum



Chile relleno



Red chile bowl



Green chile bowl

# New Mexico State University, Las Cruces



**Land grant institution, Carnegie R2 (soon to be R1)**

Comprehensive: Arts and Sciences, Education, Business, Agriculture  
Ph.D. programs in sciences, engineering, agriculture; Ag extension;

**Chile Pepper Institute**

**12,700 students** (11,000 UG, 1,700 GR), 1000 faculty

**Minority-serving**, Hispanic-serving (60% Hispanic/NA, 26% White)  
Small-town setting (111,000)

**Military-friendly** institution (Army and Air Force ROTC programs)

**Community engagement** classification  
(first-generation students, Pell grant recipients)

**Physics: BS/BA, MS, PhD degrees.** 67 UG and 39 GR students.  
**11 faculty** (HE Nuclear and Materials Physics), **2.4 M\$ expenditures.**  
**ABET-accredited BS in Physics** and BS in Engineering Physics



# Problem statement

- (1) Achieve a **quantitative** understanding of **photon absorption** and **emission** processes.
  - Our **qualitative** understanding of excitonic absorption is 50-100 years old (Einstein coefficients),
  - But **insufficient** for modeling of detectors and emitters.
- (2) How are optical processes affected by **high carrier concentrations** (screening)?
  - High carrier densities can be achieved with
    - In situ doping or
    - **ultrafast (femtosecond) lasers** or
    - high temperatures (narrow-gap or gapless semiconductors)
  - **Application:** CMOS-integrated mid-infrared camera (thermal imaging with a phone).
  - Future: How are optical processes affected by an electric field (pin diode or thin layer)?



## AVS 70th International Symposium & Exhibition Spectroscopic Ellipsometry

November 3-8, 2024 | Tampa, Florida | Call for Abstracts Deadline: **May 13, 2024**

# Application: Midwave Infrared Detectors Germanium-Tin Alloys

Intensity of Optical Absorption by Excitons

R. J. Elliott

Phys. Rev. **108**, 1384 – Published 15 December 1957

Article

References

Citing Articles (1,780)

PDF

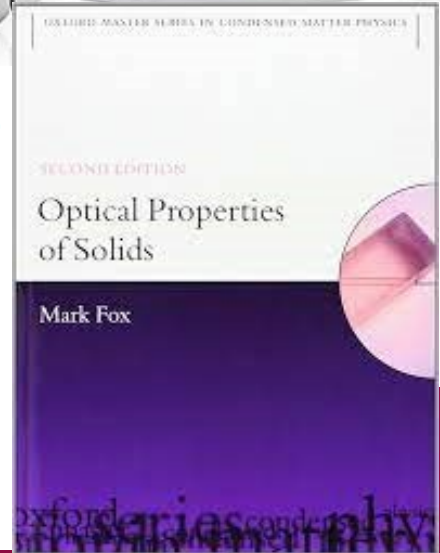
Export Citation



## ABSTRACT

The intensity of optical absorption close to the edge in semiconductors is examined using band theory together with the effective-mass approximation for the excitons. Direct transitions which occur when the band extrema on either side of the forbidden gap are at the same  $\mathbf{K}$ , give a line spectrum and a continuous absorption of characteristically different form and intensity, according as transitions between band states at the extrema are allowed or forbidden. If the extrema are at different  $\mathbf{K}$  values, indirect transitions involving phonons occur, giving absorption proportional to  $(\Delta E)^{\frac{1}{2}}$  for each exciton band, and to  $(\Delta E)^2$  for the continuum. The experimental results on  $\text{Cu}_2\text{O}$  and Ge are in good qualitative agreement with direct forbidden and indirect transitions, respectively.

Received 9 April 1957





# Ellipsometry at NMSU

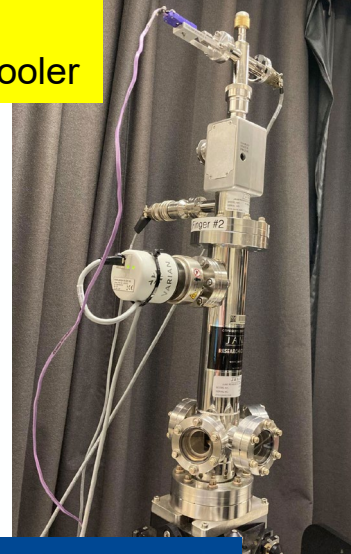
diamond windows  
closed-cycle He cooler



Ellipsometry on anything (inorganic, 3D)

- Metals, insulators, semiconductors
- Mid-IR to vacuum UV (150 nm to 40  $\mu\text{m}$ )
- **10 to 800 K, ultrafast ellipsometry**

Ellipsometry tells us a lot about materials quality (not necessarily what we want to know).



[Optical critical points of thin-film  \$\text{Ge}\_{1-y}\text{Sn}\_y\$  alloys: A comparative  \$\text{Ge}\_{1-y}\text{Sn}\_y\$  /  \$\text{Ge}\_{1-x}\text{Si}\_x\$  study](#)

440

2006

VR D'costa, CS Cook, AG Birdwell, CL Littler, M Canonico, S Zollner, ...  
Physical Review B—Condensed Matter and Materials Physics 73 (12), 125207

[Growth and strain compensation effects in the ternary  \$\text{Si}\_{1-x-y}\text{Ge}\_x\text{C}\_y\$  alloy system](#)

397

1992

K Eberl, SS Iyer, S Zollner, JC Tsang, FK LeGoues  
Applied physics letters 60 (24), 3033-3035

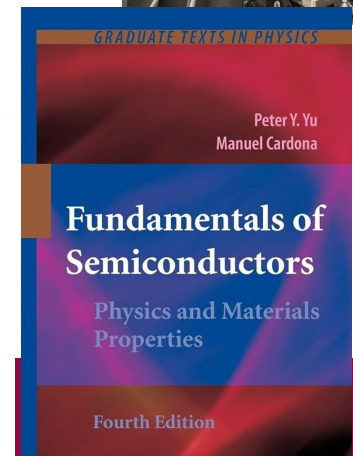
[Ge–Sn semiconductors for band-gap and lattice engineering](#)

335

2002

M Bauer, J Taraci, J Tolle, AVG Chizmeshya, S Zollner, DJ Smith, ...  
Applied physics letters 81 (16), 2992-2994

<http://femto.nmsu.edu>



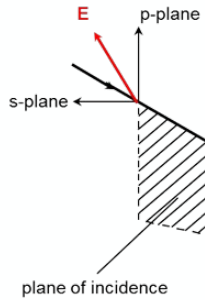
# Bandwidth considerations

- **Heisenberg uncertainty principle:**  $\Delta E \cdot \Delta t = \frac{\hbar}{2} = 330 \text{ meV} \cdot \text{fs}$
- **Attosecond spectroscopy:**  
Low spectral resolution: 33 eV for 10 as pulse duration (use x-ray pulses)  
High temporal resolution: Follow the real-space motion of electrons (and nuclei)
- **Femtosecond spectroscopy:**  
Better spectral resolution: 33 meV for 10 fs pulse length  
Minimal coherent artifacts (dephasing time smaller than pulse width)  
Temporal resolution insufficient to follow the real-space motion of electrons.  
Treat electrons and nuclei motion as **waves: Reciprocal space** (Brillouin zone of a crystal)
- **Molecular (or crystal) vibration spectroscopy (phonons):**  
Requires high spectral resolution (1 meV or better): use picosecond pulses
- **Electronic state (band structure) spectroscopy (this work):**  
Requires moderate spectral resolution (10 meV): use 30 femtosecond pulses (or longer)

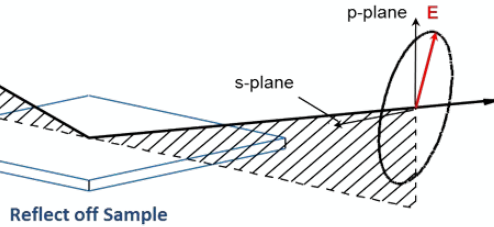


# Spectroscopic ellipsometry

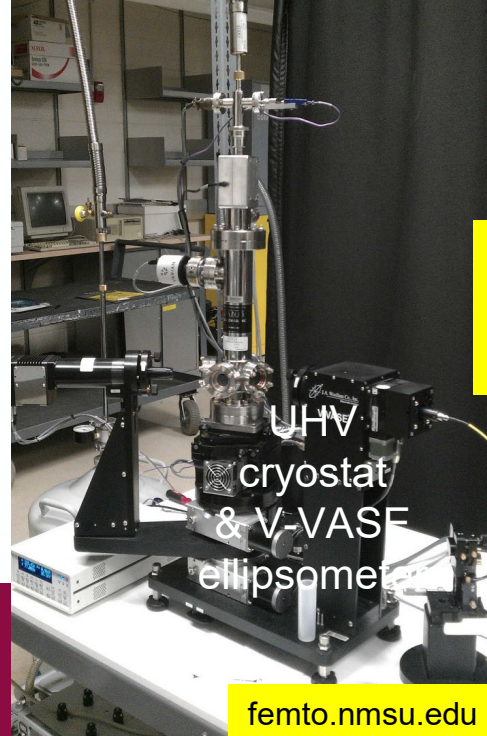
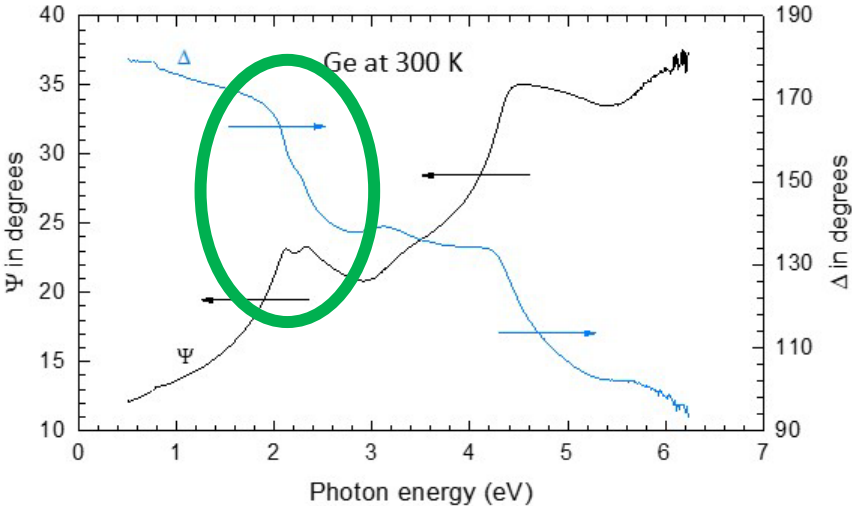
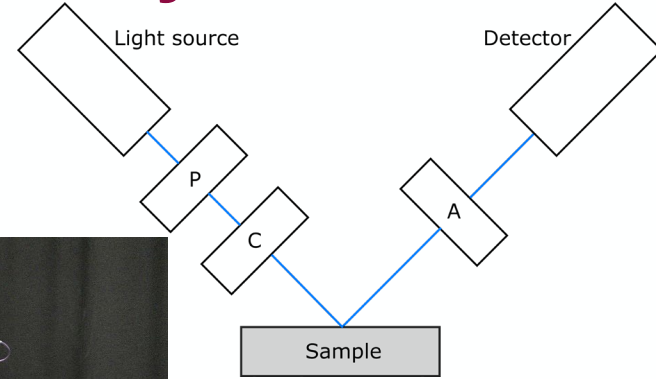
Linearly Polarized Light



Elliptically Polarized Light



Reflect off Sample



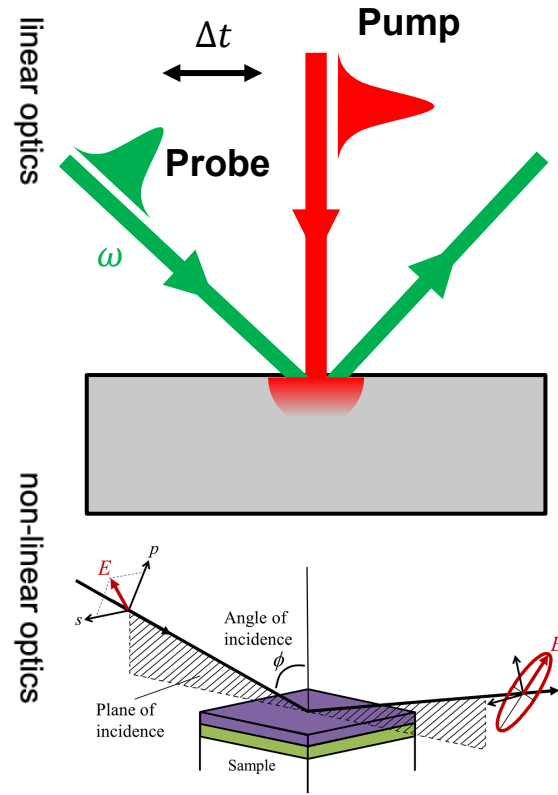
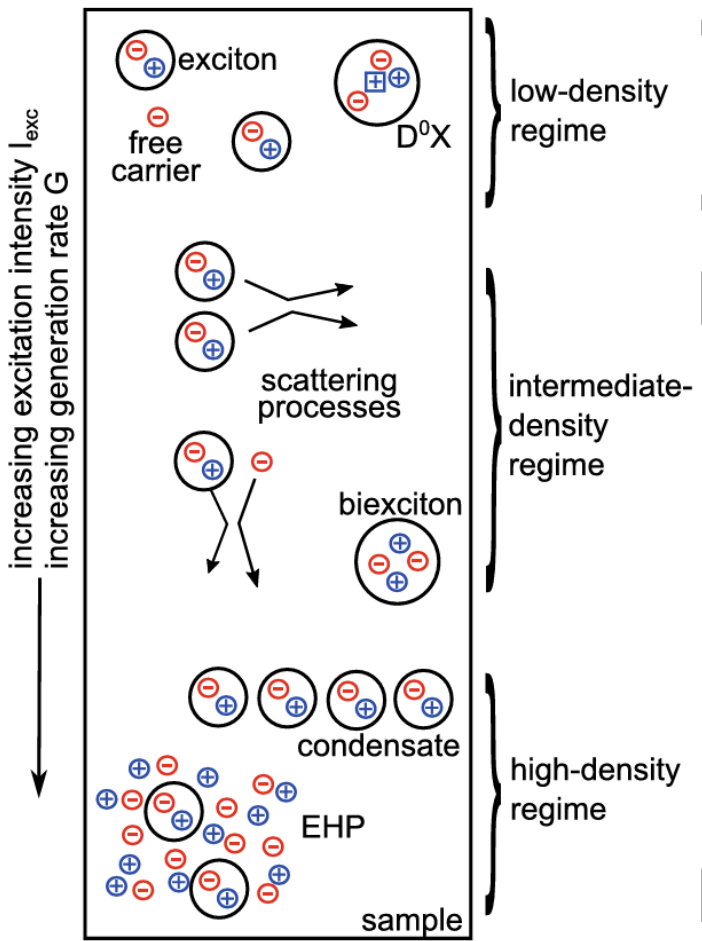
ellipsometric angles  $\psi$ ,  $\Delta$   
 refractive index  $n+ik$   
 dielectric function  $\epsilon$

$$\rho = \frac{r_p}{r_s} = \tan \Psi e^{i\Delta}$$

femto.nmsu.edu

Tompkins & Hilfiker,  
 Spectroscopic  
 Ellipsometry (2016)

# Femtosecond Pump-Probe Ellipsometry

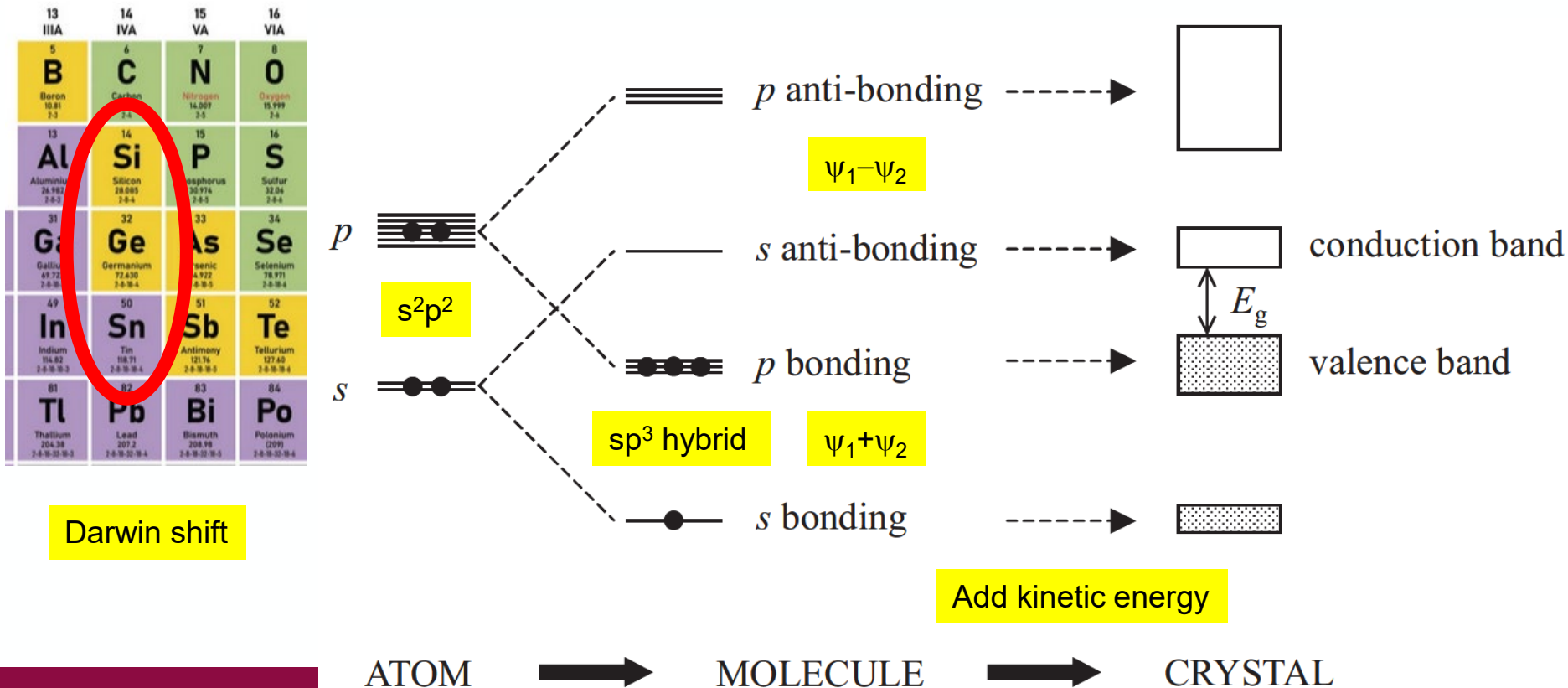


Non-linear effects in germanium induced by photoexcited carriers:

- Screening (many-body)
- Carrier-carrier scattering.
- Carrier-phonon scattering.
- Intervalley scattering.
- Momentum and energy relaxation of hot carriers.



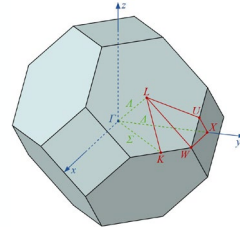
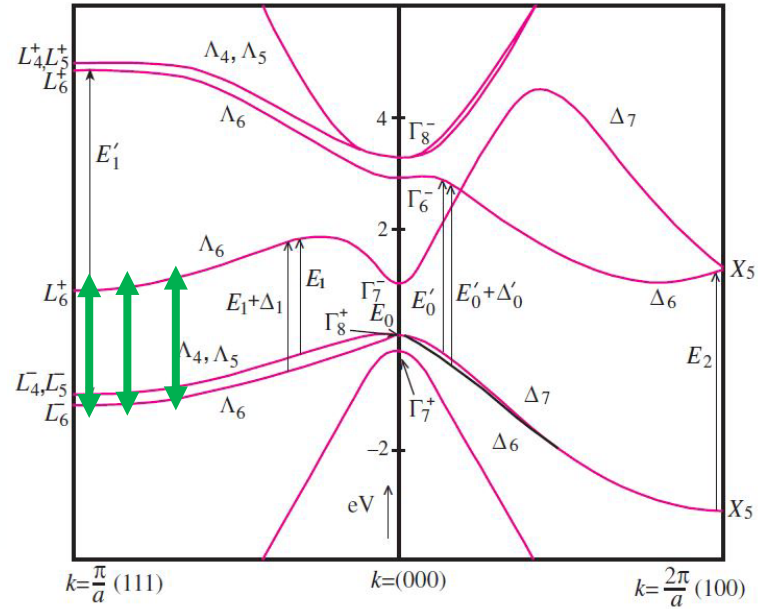
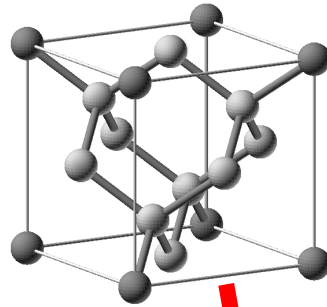
# Ge Band structure: where did this come from?



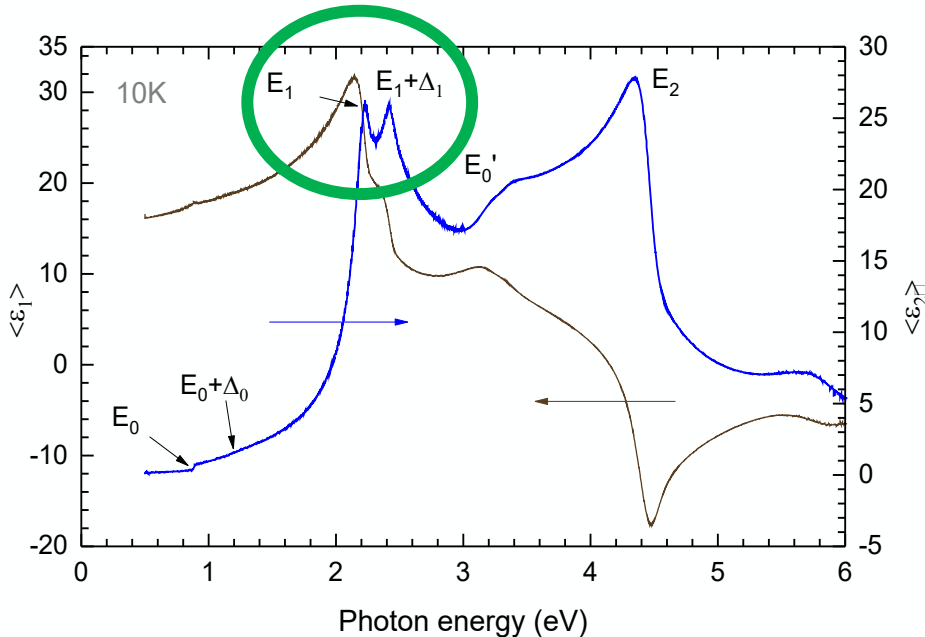
Works well for Ge, GaAs, etc.

# Critical points in the dielectric function of Ge

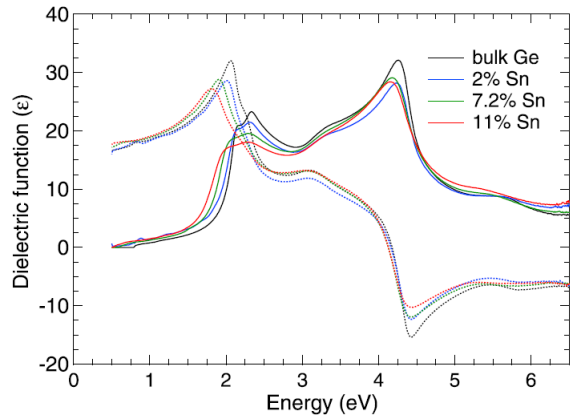
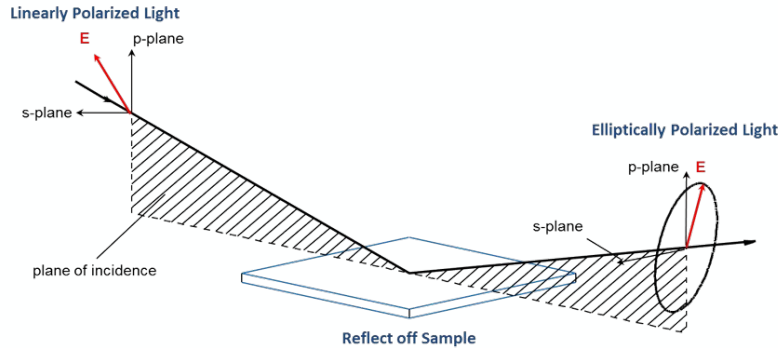
- Peaks in the dielectric function
- Due to interband transitions from valence to conduction band (electron-hole pairs)



$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_{n,\vec{k}}(\vec{r})$$

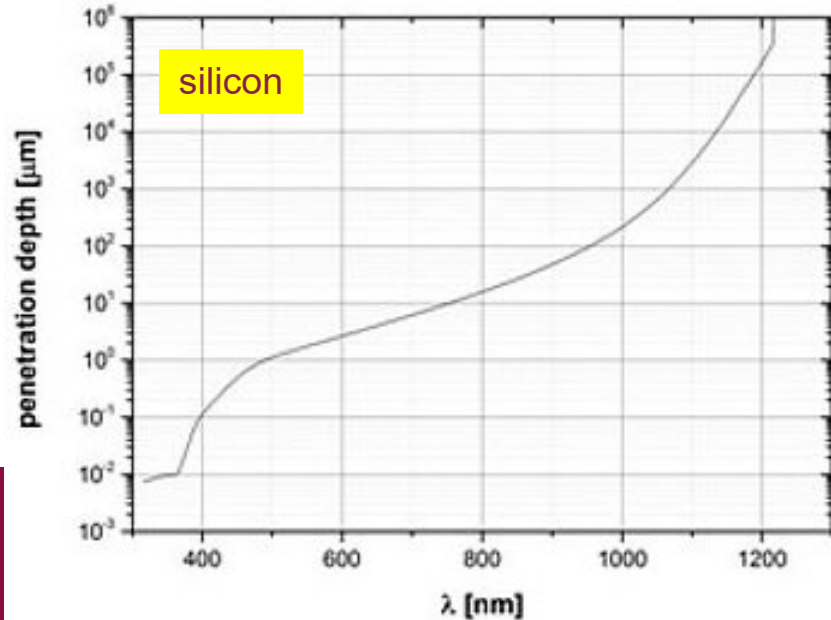


# Penetration depth



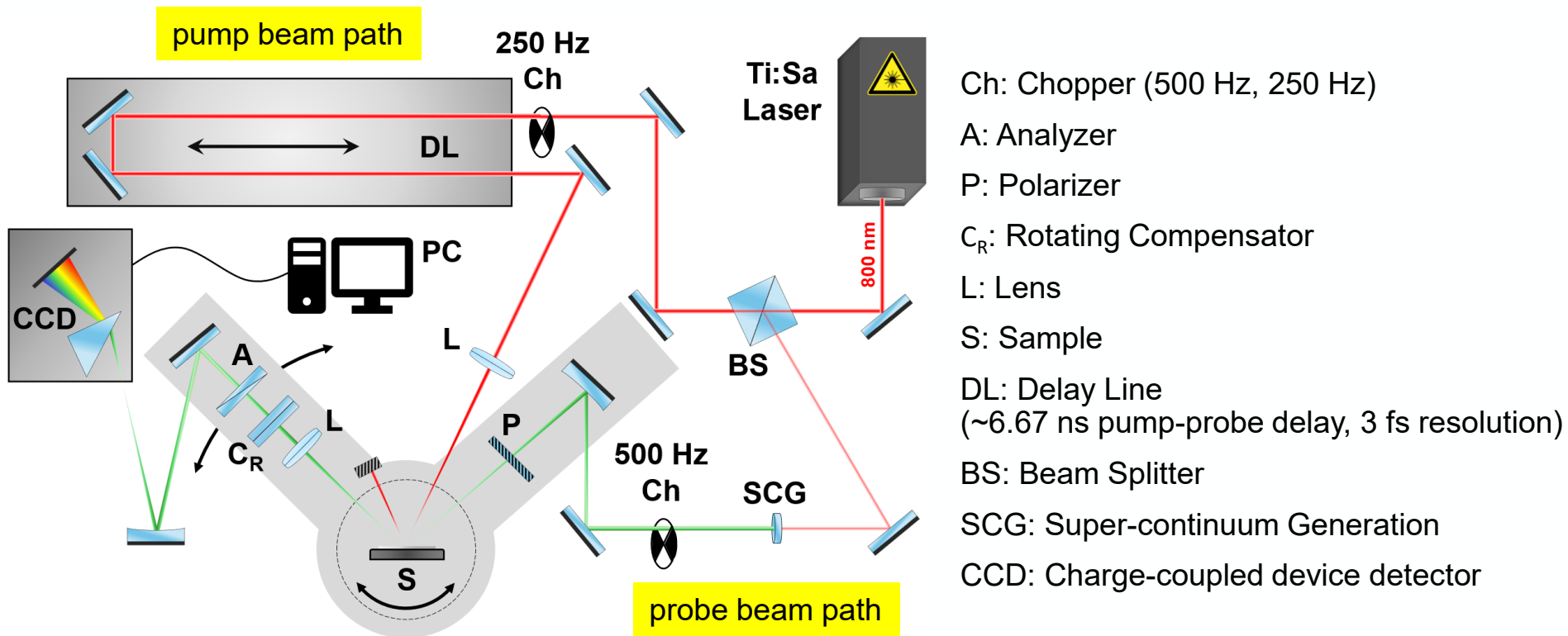
The penetration depth is the inverse of the absorption coefficient (measured with spectroscopic ellipsometry).

Below the direct band gap (0.8 eV for Ge, 3.4 eV for Si), the penetration depth is very large (many micrometers). The penetration depth is smallest at the  $E_2$  peak in the UV, about 10 nm.





# Experimental setup: pump-probe ellipsometry



# Experimental setup: pump-probe ellipsometry

35 fs, 800 nm, 1 kHz

pump beam path

250 Hz  
Ch

Ti:Sa  
Laser

Ch: Chopper (500 Hz, 250 Hz)

A: Analyzer

P: Polarizer

C<sub>R</sub>: Rotating Compensator

L: Lens

S: Sample

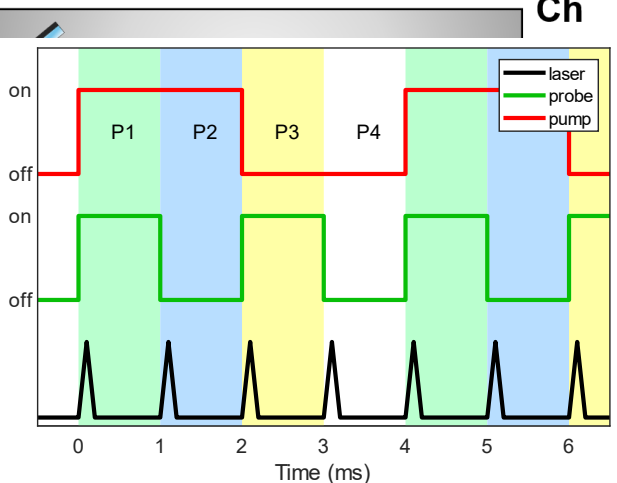
DL: Delay Line

(~6.67 ns pump-probe delay, 3 fs resolution)

BS: Beam Splitter

SCG: Super-continuum Generation

CCD: Charge-coupled device detector



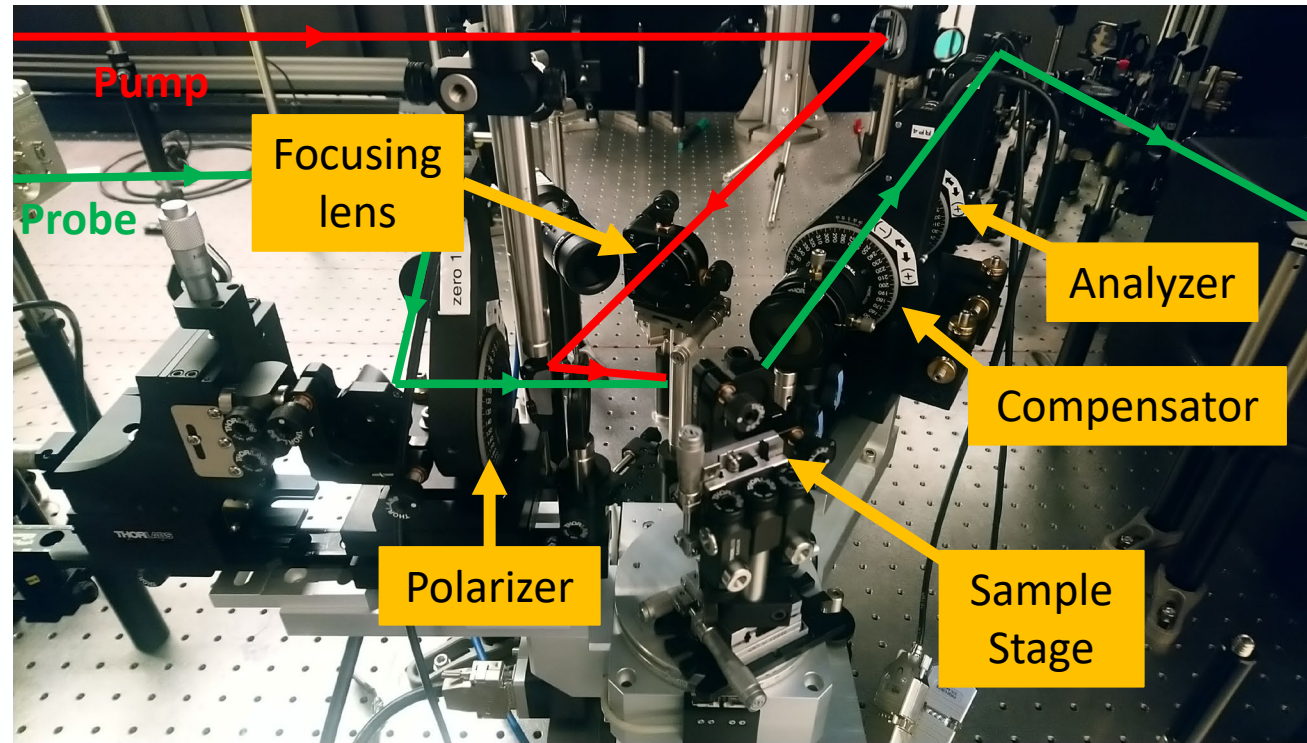
$$\frac{\Delta R^p(E, \Delta t)}{R^0(E)} = \frac{I_{P1} - I_{P2}}{I_{P3} - I_{P4}} - 1$$

probe beam path

500 Hz  
Ch

SCG

# Set-up: Femtosecond pump-probe ellipsometry



## Rotating compensator ellipsometer:

Compensator was rotated in steps of  $10^\circ$  for a total of 55-65 angles.

Probe beam of 350-750 nm at  $60^\circ$  incidence angle.

P-polarized pump beam: 35 fs pulses of 800 nm wavelength at 1 kHz repetition rate.

Delay time from -10 to 50 ps.

Time resolution of about 500 fs.



# Femtosecond pump-probe ellipsometry at ELI ALPS

Pump

The femtosecond ellipsometer at ELI ALPS is similar, with some differences:

- Better time resolution (10 fs)
- Smaller spot size (10  $\mu\text{m}$ )
- **Pump and probe with the same laser beam**
- No white light continuum probe
- Simpler setup

Rotating compensator

ellipsometer:

... was rotated in steps  
total of 55-65 angles.

... of 350-750 nm at 60°  
angle.

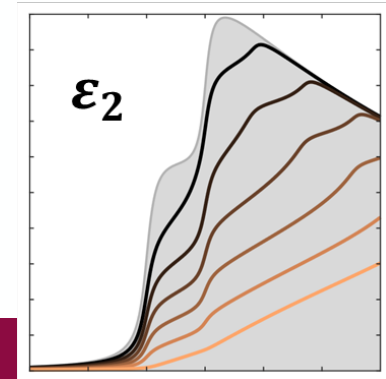
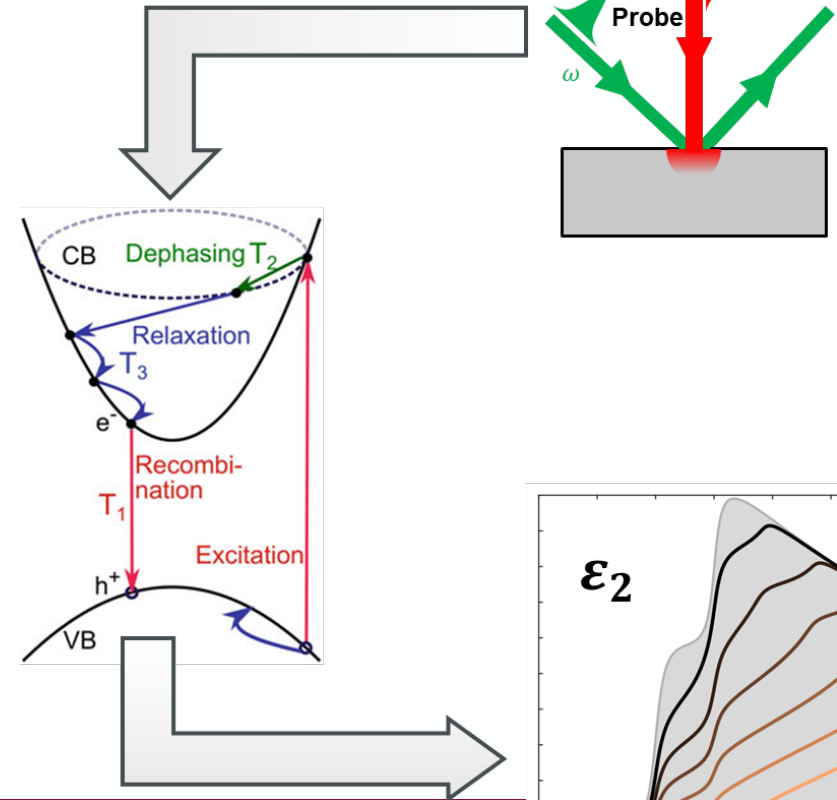
... pump beam: 35 fs  
... nm wavelength at 1  
... n rate.

... om -10 to 50 ps.

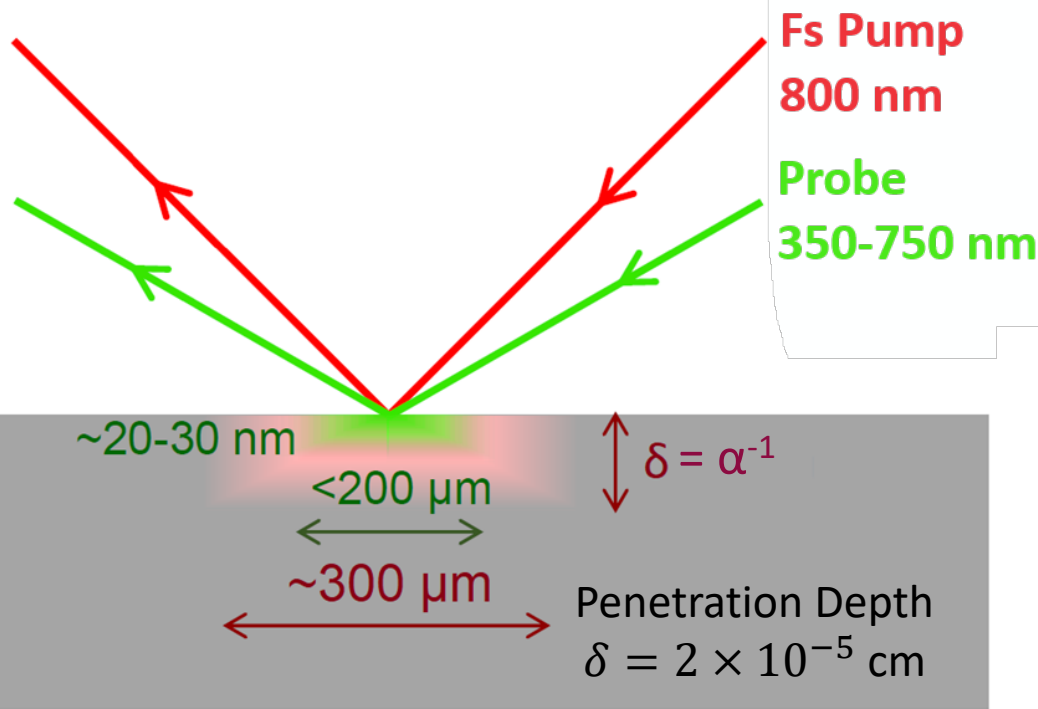
Time resolution of about 500 fs.

# Outline

- Introduction:
- Experimental set up and procedure.
- Pump-pulse absorption and charge carrier density.
- Raw data.
- Ultrafast dynamics:
- Scattering processes and carrier statistics.
- Preliminary results.
- Modeling additional absorption processes.
- Conclusion & future work



# Absorption of pump energy: charge carrier density



Number of Incident Photons

$$N = \frac{E_{\text{pulse}}}{E_{\text{photon}}} = \frac{P/f}{\omega \hbar}$$

Absorbed Photons

$$N_{\text{absorbed}} = (1 - R_P(\theta, \lambda))N$$

Charge Carrier Density

$$\frac{N_{\text{absorbed}}}{V} = \frac{N_{\text{absorbed}}}{A\delta}$$



# Absorption of pump energy: charge carrier density

Charge Carrier Density						
(Orientation) Pump Intensity	Bulk Ge (111)	Bulk Ge (110)	Bulk Ge (100)	nDoped Ge film*	GeSn Film (10% Sn)	GeSn Film (18% Sn)
Pump Power (mW) Pulse Energy ( $\mu\text{J}$ )	13	14	3	2.5	2	5
Pump Beam Diameter ( $\mu\text{m}$ )	295	295	180	261	269	517
Charge Carrier Density ( $\text{cm}^{-3}$ )	$3.42 \times 10^{21}$	$3.69 \times 10^{21}$	$3.16 \times 10^{21}$	$5.93 \times 10^{20}$	$7.62 \times 10^{20}$	$1.31 \times 10^{20}$

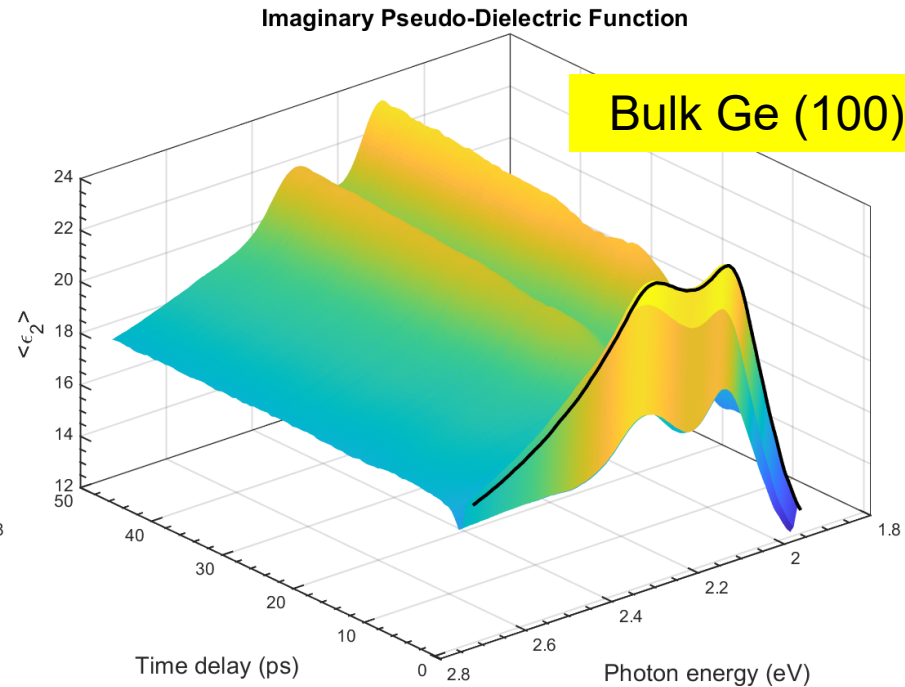
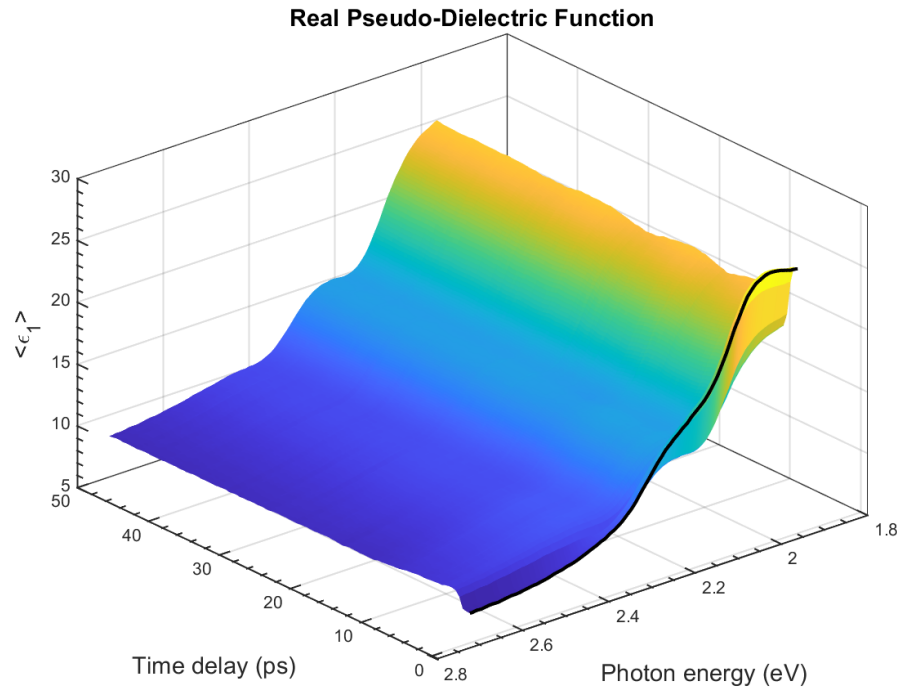
Excitation density just below the damage threshold.

**Carrier concentration of  $3 \times 10^{21} \text{ cm}^{-3}$  is not physical.**

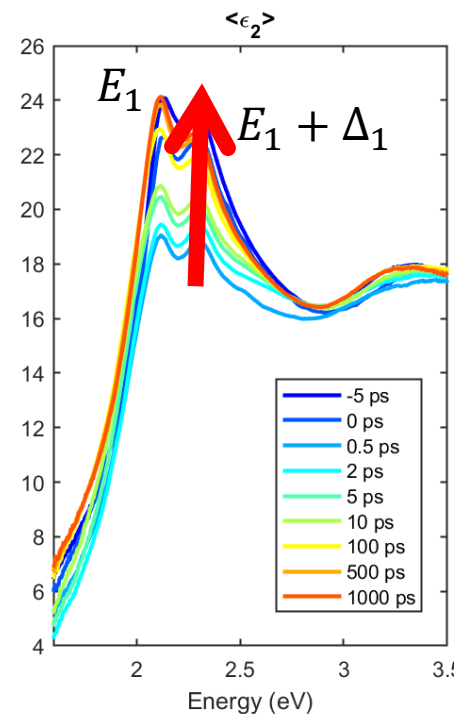
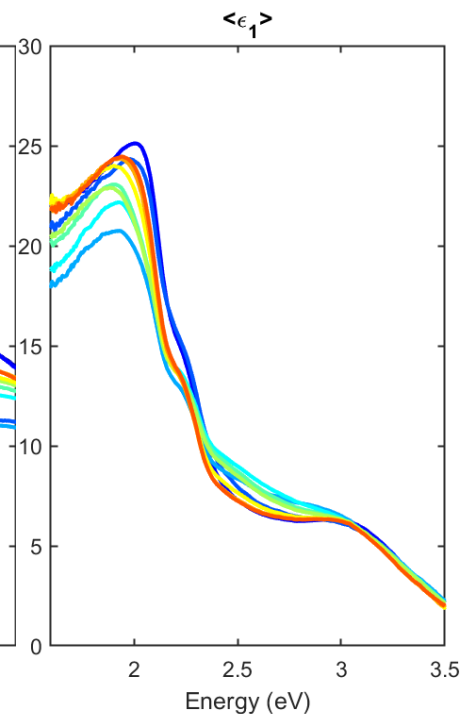
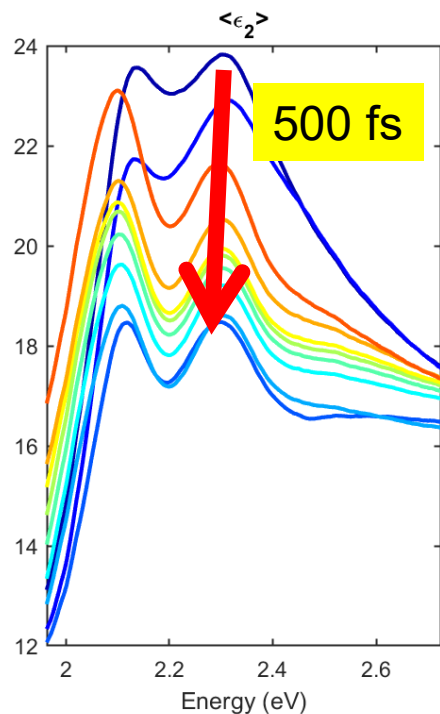
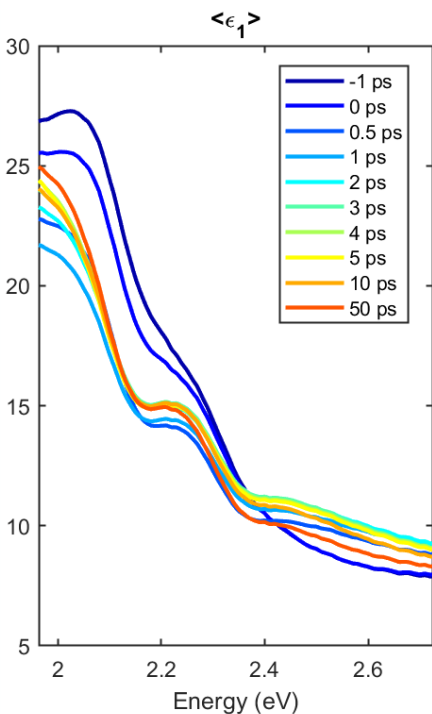
The absorption is already bleached by the pump pulse.

\* Doping:  $n = 1.05 \times 10^{20} \text{ cm}^{-3}$

# Pseudo-dielectric constant as function of delay time



# Pseudo-dielectric constant as function of delay time



Rapid decrease of  $\epsilon$  within first 500 fs.

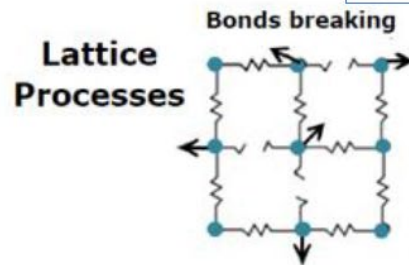
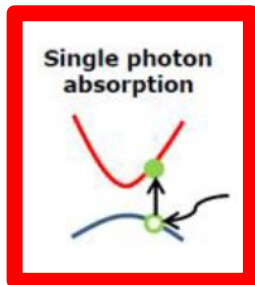
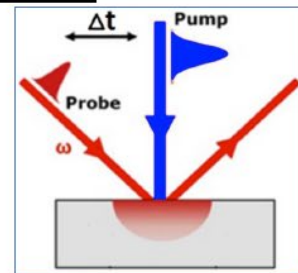
Recovery takes 1 ns or longer.



# What is happening on these time scales ?

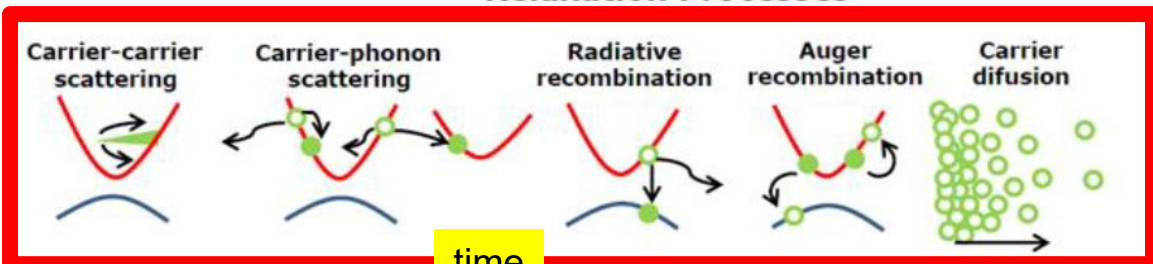


Shirly Espinoza

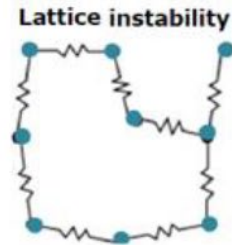


Excitation density below damage threshold.

## Relaxation Processes



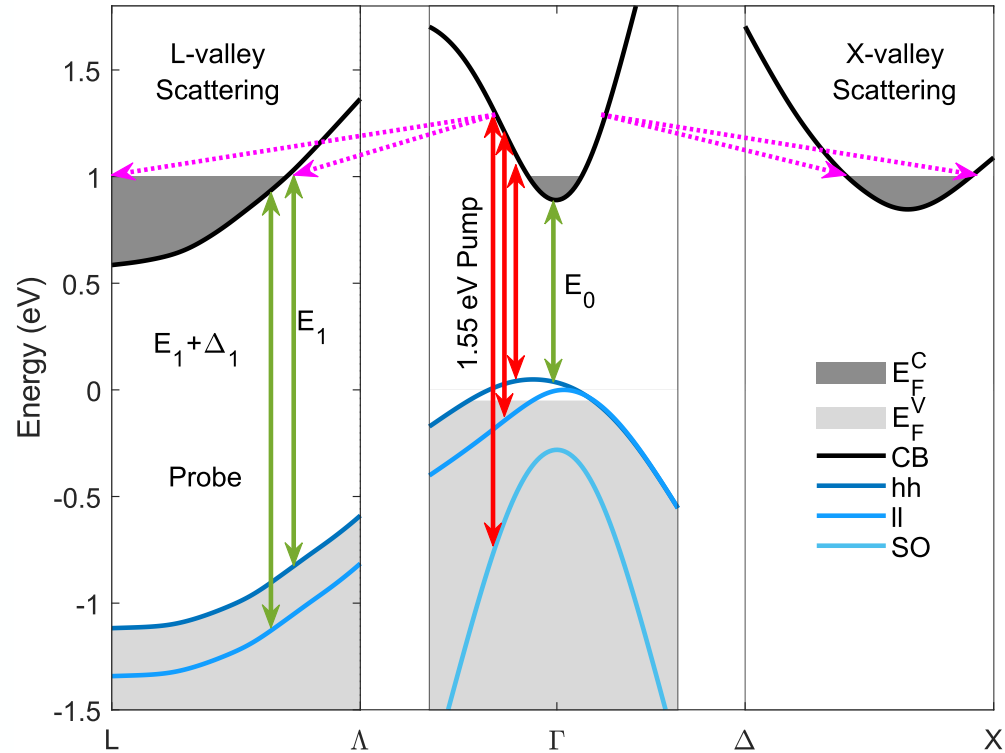
time



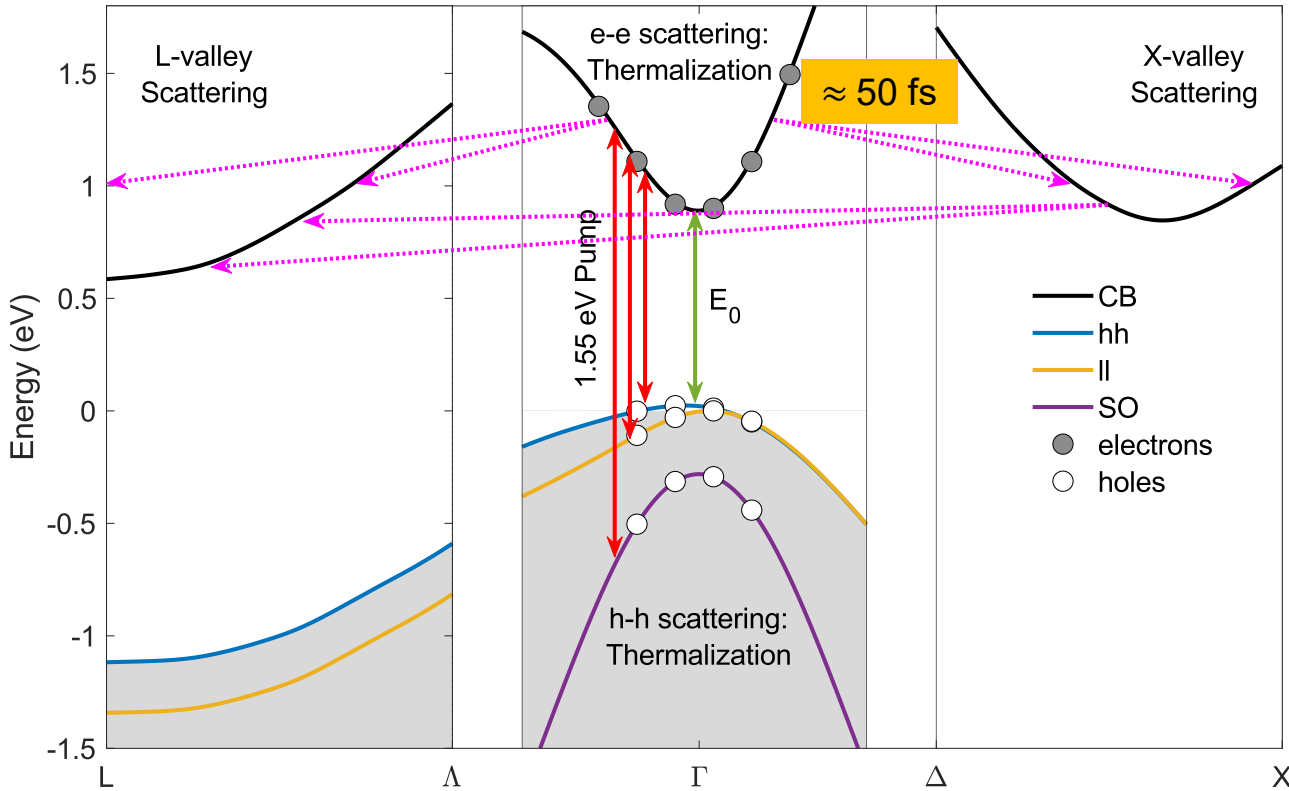
# Interband transitions in Ge

- Generation of hot electrons in the  $\Gamma$ -valley with excess energy  $\varepsilon$  ( $\approx 0.7$  eV).
- Momentum relaxation and thermalization of hot electrons ( $< 100$  fs).
- Energy relaxation by intervalley scattering (GaAs:  $\Gamma$  to X: 50 fs).
- Optical functions change as a function of the density and energy of the electrons.
- Many-body phenomena affect the band transition energy:

$$\Delta E = \Delta E_{\text{str}} + \Delta E_{\text{BM}} + \Delta E_{\text{BGR}}$$



# Carrier relaxation (<100 fs): Energy transfer to lattice



$$\varepsilon = \frac{\hbar\omega_{\text{pump}} - E_0}{1 + m_e/m_h}$$

Plasma temperature:

$$T_C \approx \frac{\varepsilon}{3k_B}$$

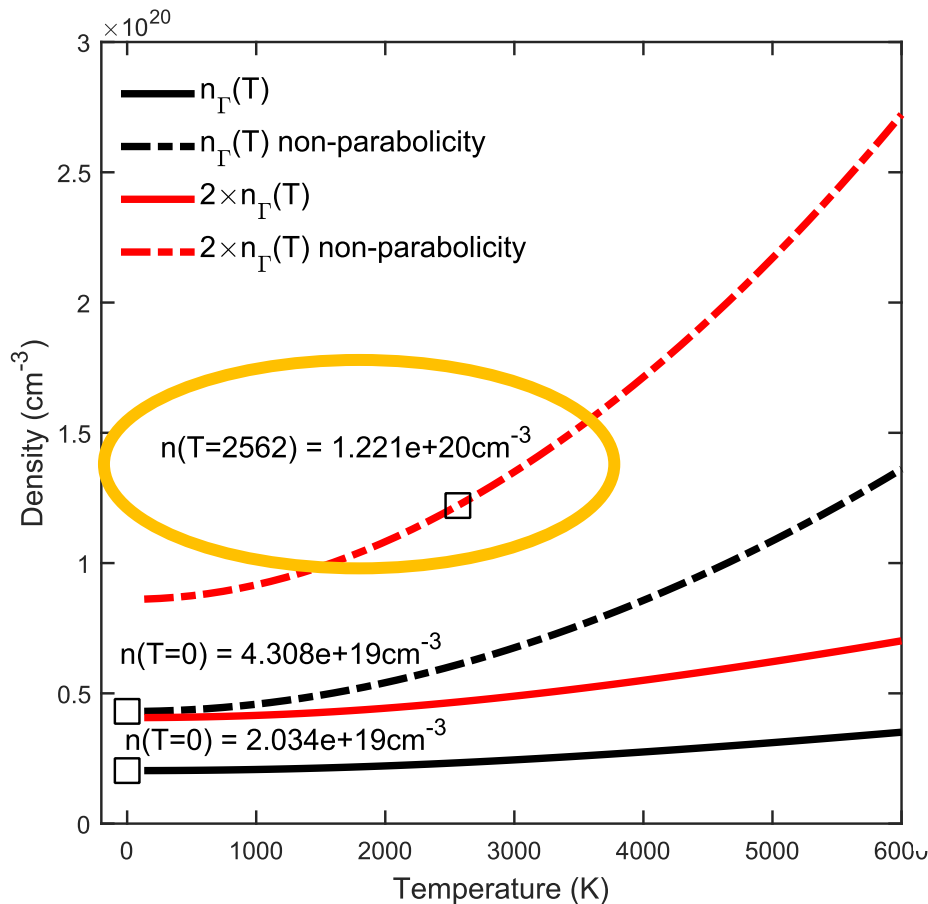
$$\varepsilon = 0.7 \text{ eV}$$

$$T = 2500 \text{ K}$$

Intervalley scattering is more efficient than intravalley scattering.

Stefan Zollner, Sudha Gopalan, and Manuel Cardona, Effective **deformation potentials** in the description of time-resolved and hot-electron luminescence, *Solid-State Commun.* **76**, 877-879 (1990)

# Electron concentration from density of states



To avoid bleaching of the absorption, the chemical potential  $\mu$  cannot be larger than the excess electron energy  $\varepsilon$ . Assume

$$\mu = \varepsilon + E_0$$

$$n_{\Gamma}(T) = \frac{1}{4} \left( \frac{2m_{e,\Gamma} k_B T}{\pi \hbar^2} \right)^{3/2} F_{1/2} \left( \frac{\varepsilon}{k_B T} \right)$$

Calculate maximum electron concentration with Fermi-Dirac statistics:

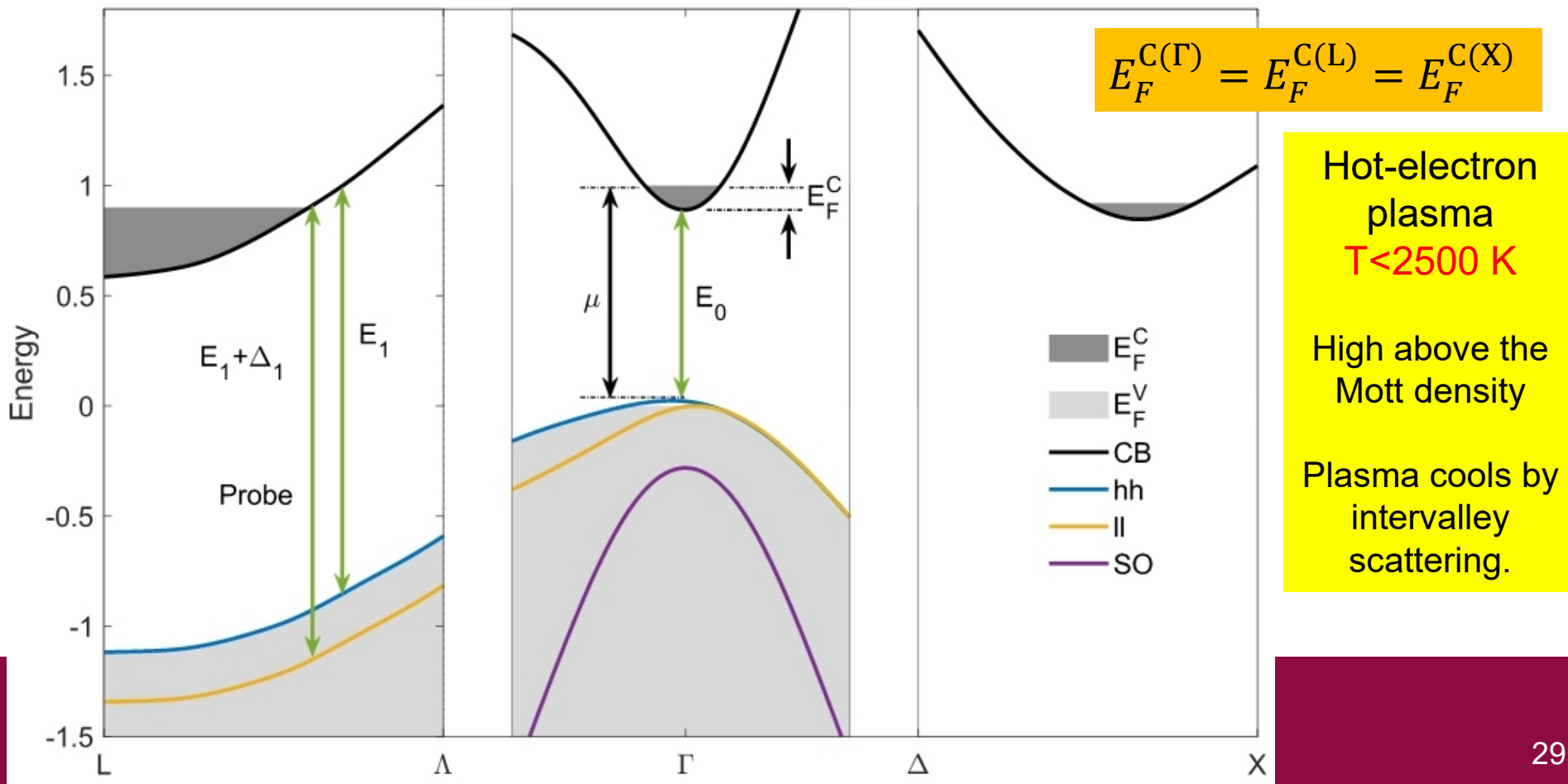
**n cannot be more than  $10^{20} \text{cm}^{-3}$ .**

High above the Mott density ( $10^{17} \text{cm}^{-3}$ ).

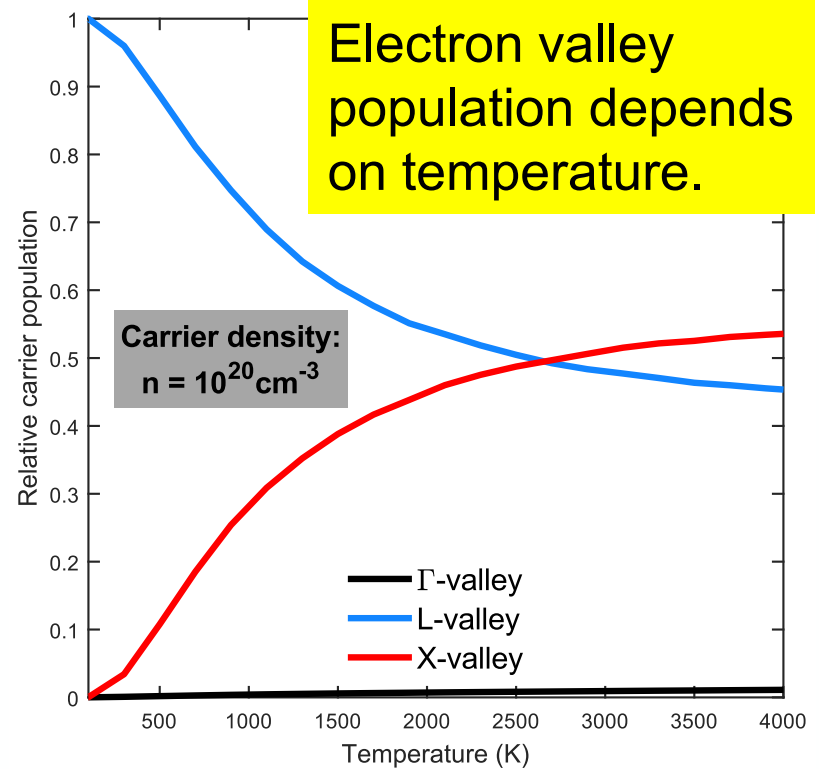
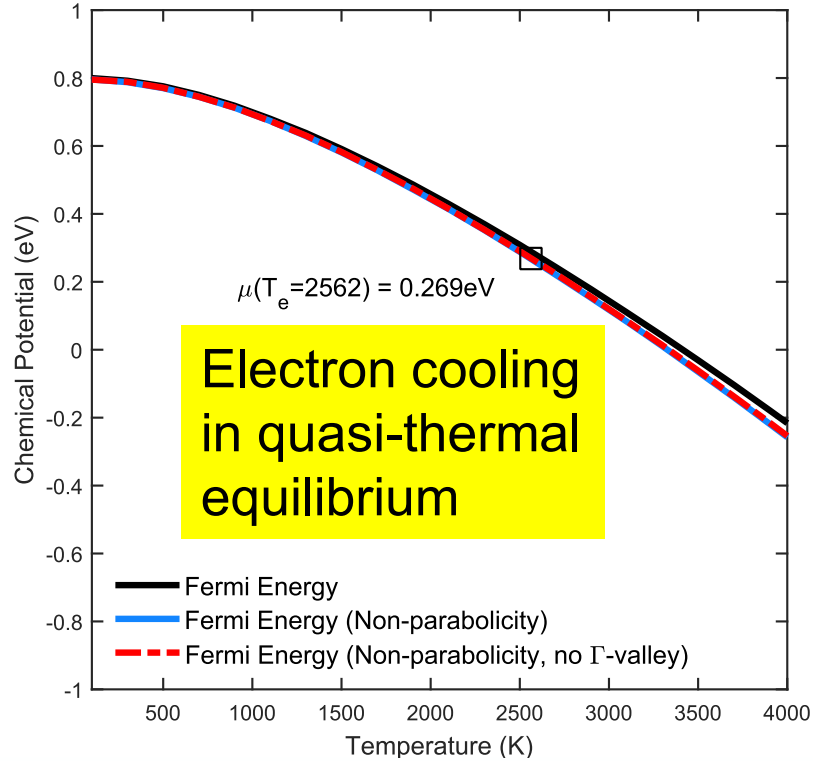
Consider density of states with conduction band non-parabolicity from k.p theory.



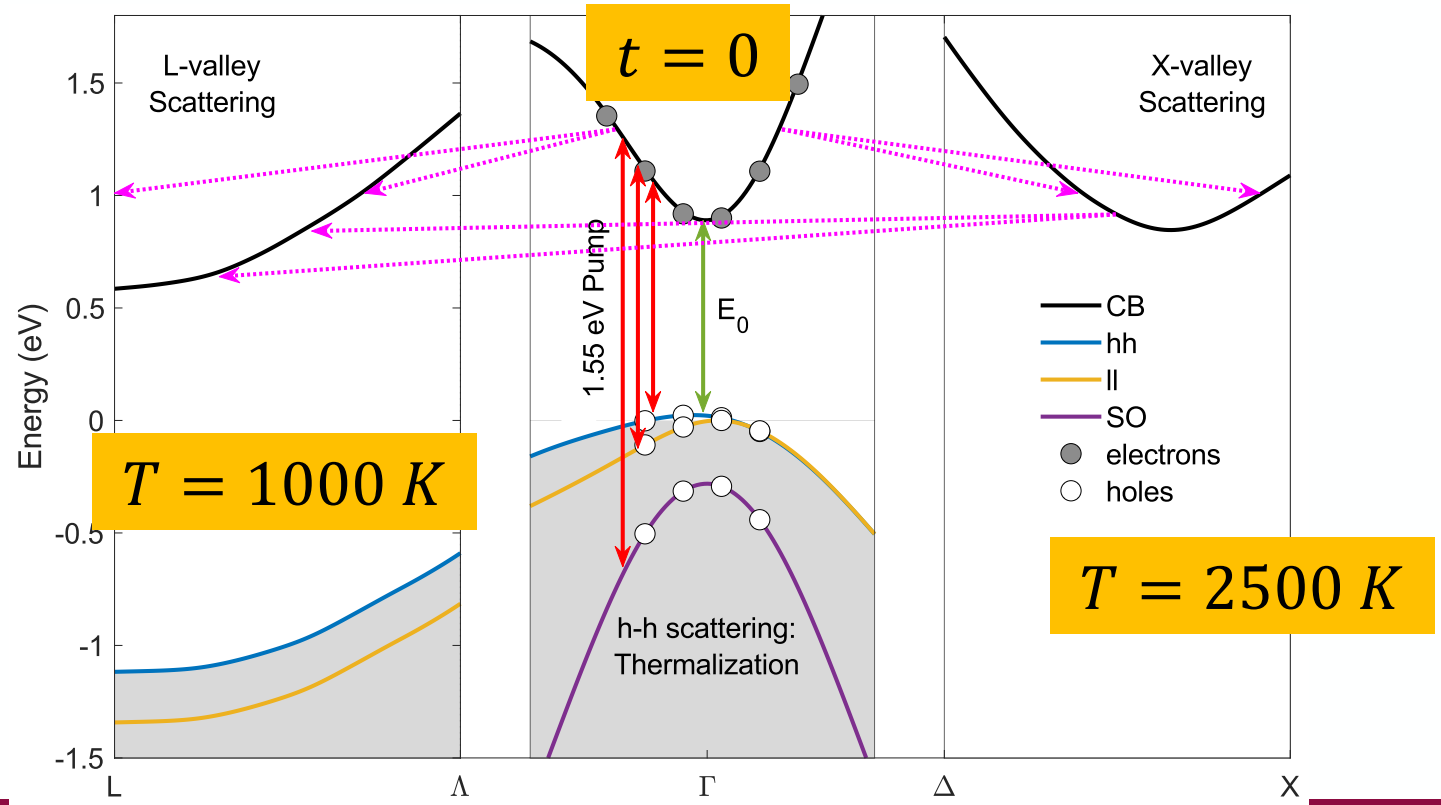
# Thermal equilibrium between valleys (>500 fs)



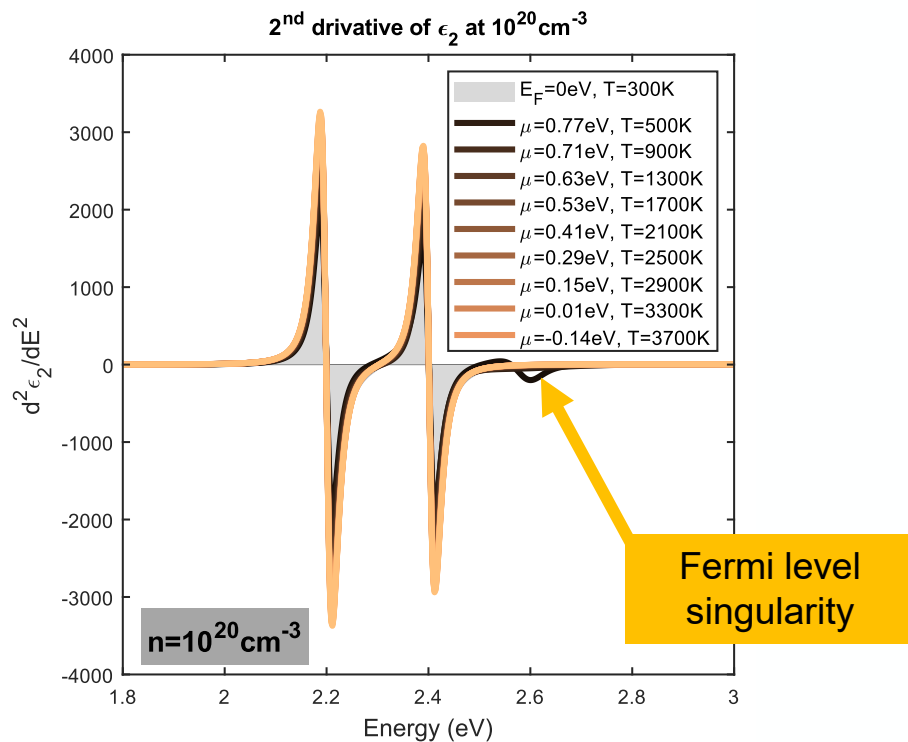
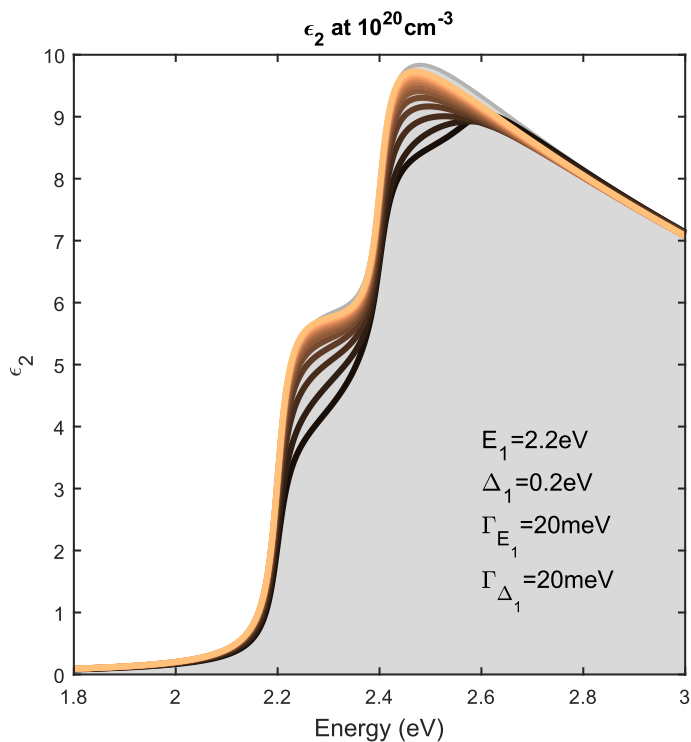
# Electrons cooling by intervalley scattering (>500 fs)



# Electrons cooling by intervalley scattering (>500 fs)



# Band-filling model for transient dielectric function



$$\epsilon_2(E) = \frac{2e^2 \bar{P}^2 \mu_{\perp}}{3\pi \epsilon_0 m^2 E^2} H(E - E_1) \int_{-k_{\max}}^{k_{\max}} 1 - f[E_c(E, k_z^2)] dk_z$$

Xu, JAP **125**, 085704 (2019).  
 Xu, PRL **118**, 267402 (2017).



# Band-filling model for transient dielectric function

$$\varepsilon_2(E) = \frac{2e^2 \bar{P}^2 \mu_{\perp}}{3\pi \varepsilon_0 m^2 E^2} H(E - E_1) \int_{-k_{\max}}^{k_{\max}} 1 - f[E_c(E, k_z^2)] dk_z$$

Introduce broadening  $\Gamma$  of interband transitions.

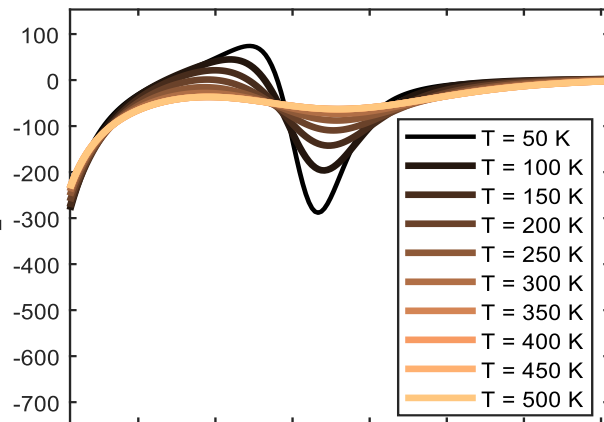
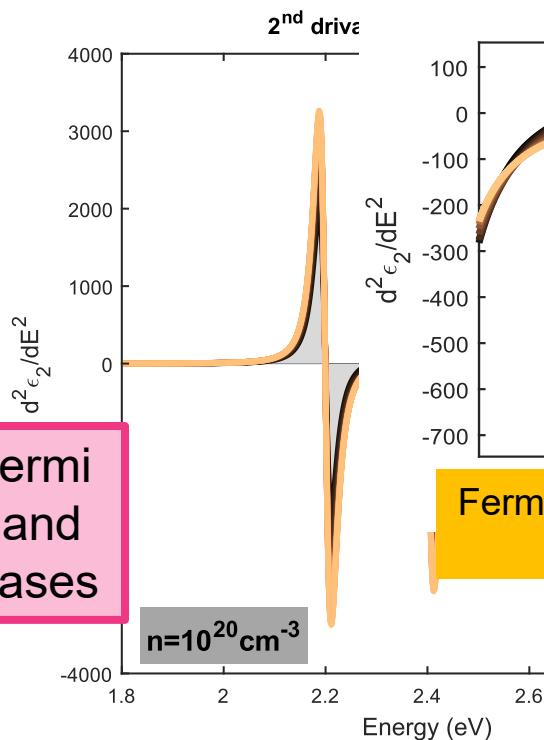
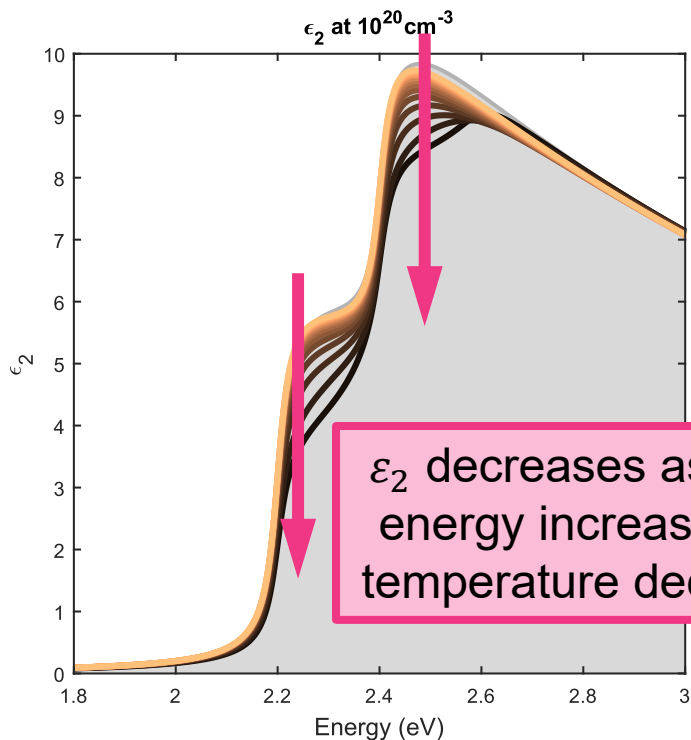
Integration over Fermi-Dirac distribution yields Fermi integral  $F_{-1/2}$ .

$$\begin{aligned} & \varepsilon_2(E, \Gamma, \mu, T) \\ &= \frac{1}{\pi} \text{Im} \left\{ \ln \left[ \frac{2(E_1 - i\Gamma - E)}{E_1 - i\Gamma} \right] \right\} \frac{4k_{\max} e^2 \bar{P}^2 \mu_{\perp}^{(E_1)}}{3\pi \varepsilon_0 m^2 E^2} \left\{ 1 - \left( \frac{k_T}{2k_{\max}} \right) F_{-1/2} \left[ \frac{\mu - (E - E_1) \frac{\mu_{\perp}}{m_{\perp}}}{k_B T} \right] \right\} \end{aligned}$$

Fixed parameters: Band gap  $E_1$ , momentum matrix element  $P$ , effective masses  $m_i$ ,  $\mu_i$

Adjustable parameters: Broadening  $\Gamma$ , chemical potential  $\mu$ , temperature  $T$ , wave vector range  $k_{\max}$

# Band-filling model for transient dielectric function



Fermi level singularity more prominent as temperature decreases.

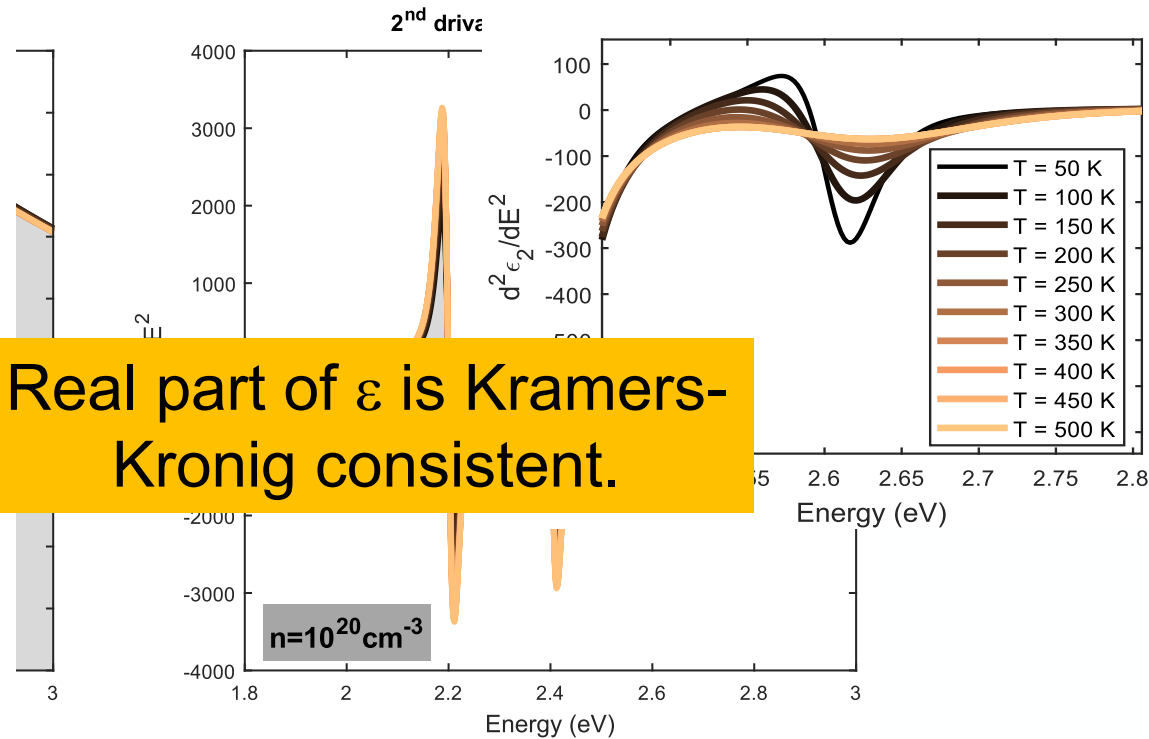
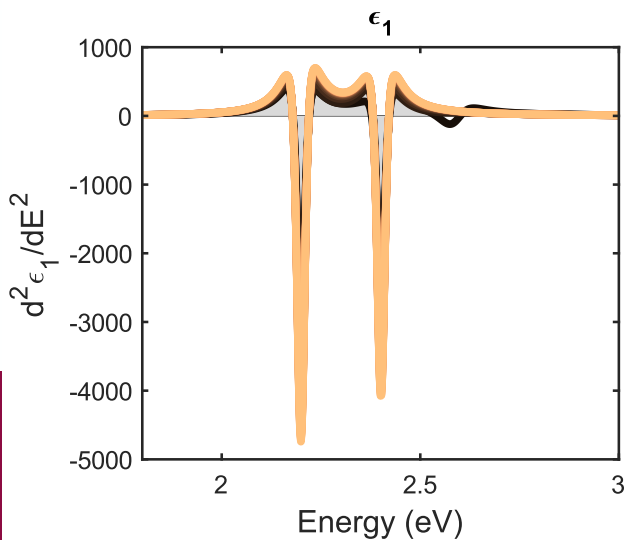
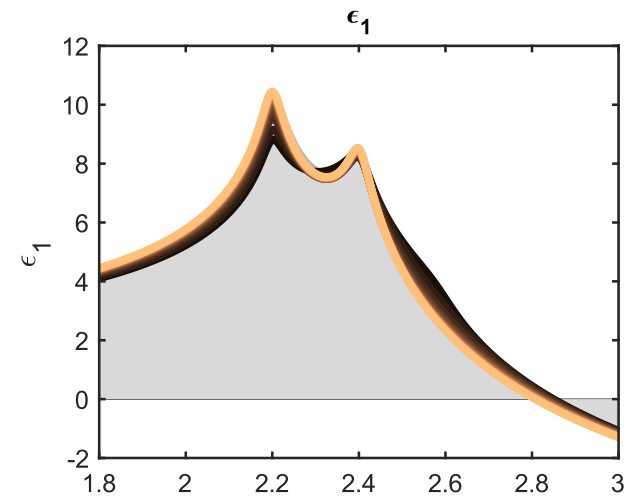
No Burstein-Moss shift for 2D CP:

$$\Delta E_{\text{BM}} = 0$$

$$\epsilon_2(E) = \frac{8e^2 \bar{P}^2 \mu_{\perp}}{3m^2 E^2} H(E - E_1) \int_{-k_{\text{max}}}^{k_{\text{max}}} 1 - f[E_c(E, k_z^2)] dk_z$$

Xu, JAP **125**, 085704 (2019).  
Xu, PRL **118**, 267402 (2017).

# al for transient dielectric function

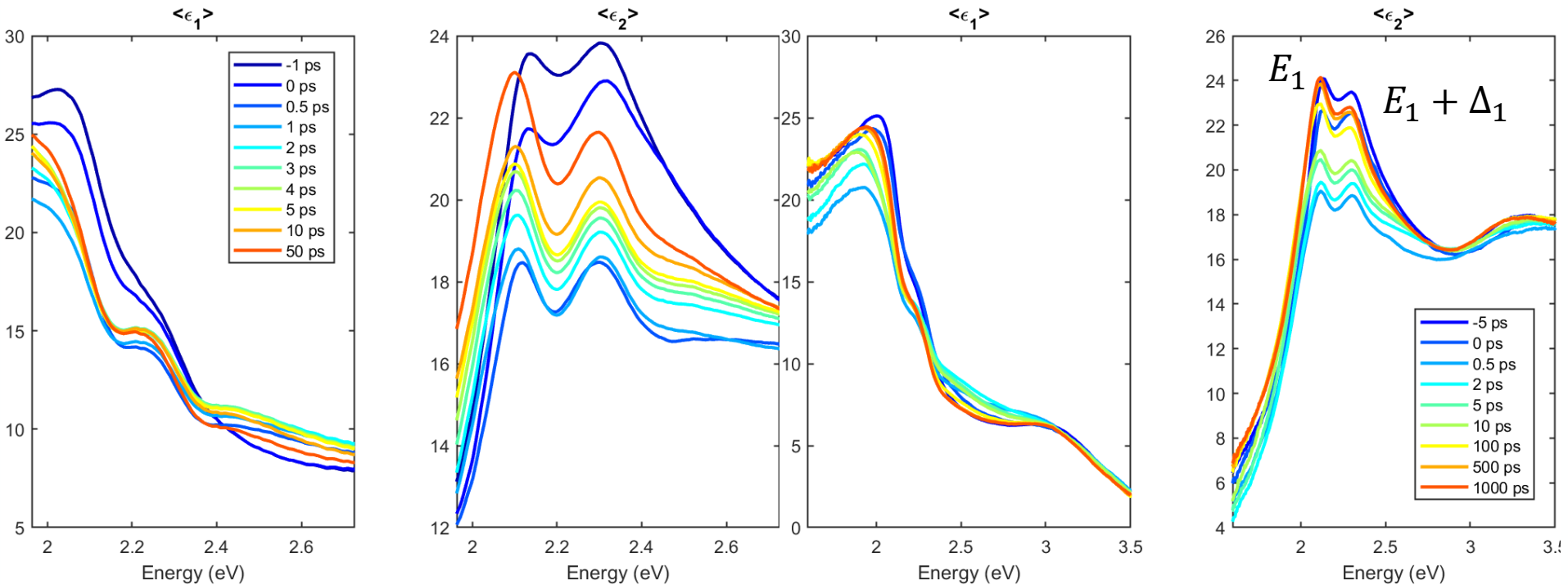


Real part of  $\epsilon$  is Kramers-Kronig consistent.

ure.

Xu, JAP **125**, 085704 (2019).  
Xu, PRL **118**, 267402 (2017).

# Pseudo-dielectric constant as function of delay time



No quantitative agreement.  
Need to include excitonic effects.



# What have we learnt so far ?

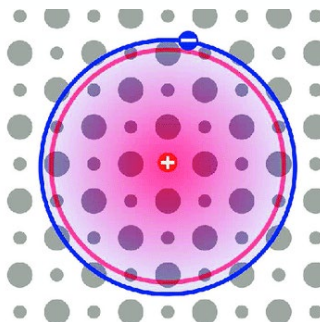
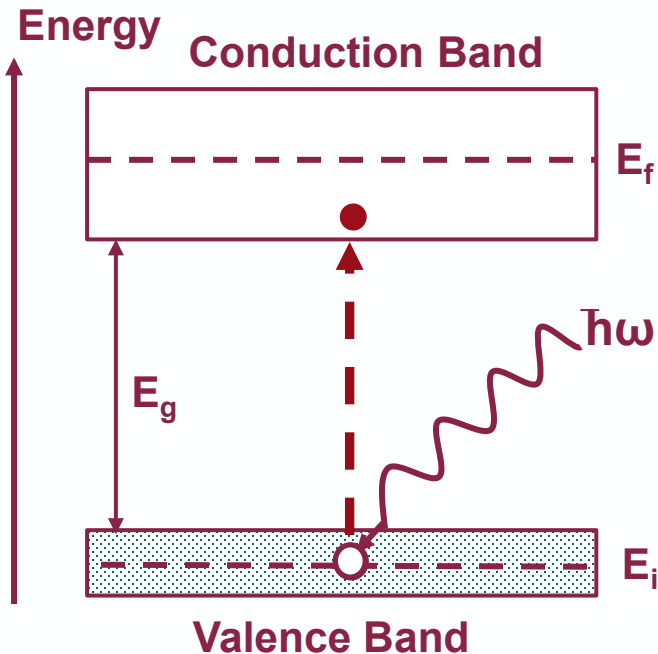
- Calculated electron-hole concentration (about  $10^{20} \text{ cm}^{-3}$ ) from Fermi-Dirac statistics.
- Electrons initially in  $\Gamma$ -valley, very hot electron plasma (2500 K)
- Within 50 fs, most electrons (>50%) scatter to the X-valley (large density of states).
- Electrons cool by intervalley scattering.

When  $T < 1000 \text{ K}$ , most electrons are in the L-valley

L-electrons are observable by bleaching the absorption of the probe pulse.

- Theory predicts a reduction of  $\varepsilon_2$  due to band filling (Pauli blocking) by about 20%.
- **A reduction by 25% is observed in the experiment, but the amplitude and line shape are wrong.**
- **What is missing?**
- L-valley absorption is enhanced by excitons.
- Excitonic (Sommerfeld) enhancement is screened by high electron density.
- Next, we need to talk about **excitons**.

# Exciton concept: Bound Electron-Hole Pair



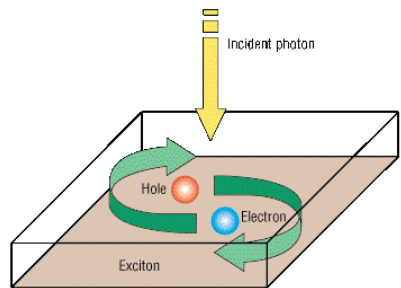
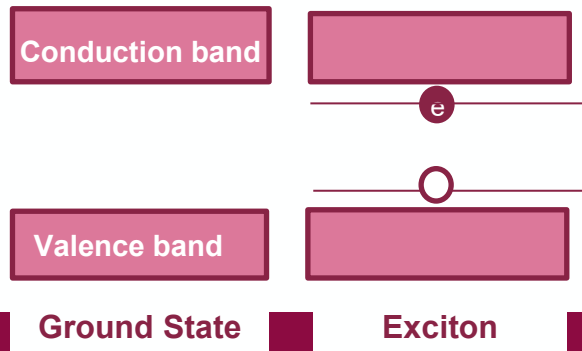
- Large radius (larger than atomic spacing)
- Weakly bound

Bohr model for exciton

## Excitons in semiconductors

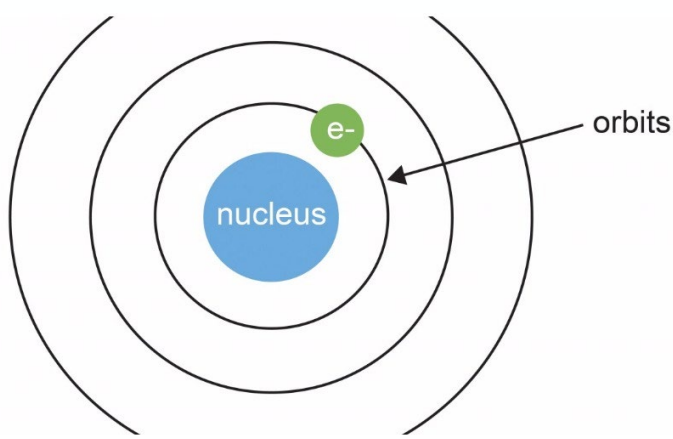
	Excitonic Radius(Å)	Lattice Constant(Å)	Excitonic Binding Energy (meV)	$R$
GaAs	130	5.6532	4.2	
SrTiO <sub>3</sub>	62.5	3.9050	20	
GaP	50	5.4505	21	
ZnO	20	a=3.2500, c=5.2040	60	

### Semiconductor Picture



Shape the Future.

# Bohr model for free excitons



Electron and hole form a bound state with binding energy.

$$E(n) = -\frac{\mu}{m_0} \frac{1}{\epsilon_r^2} \frac{R_H}{n^2} = -\frac{R}{n^2}$$

$R_H = 13.6$  eV Rydberg energy.  
QM mechanical treatment easy.

1. Reduced electron/hole mass

(**optical mass**)

$$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_h}$$

2. **Static screening** with static dielectric constant  $\epsilon_r$ .

3. **Exciton radius:**

$$a_n = \frac{m_0}{\mu} \epsilon_r n^2 a_H$$

$$a_H = 0.53 \text{ \AA}$$

4. Excitons **stable** if  $R \gg kT$  ????

5. Exciton **momentum** is zero.

6. **Exciton enhancement important even if  $R \ll kT$ .**

# Sommerfeld enhancement

Excitonic Rydberg energy

$$R = \frac{\mu}{m_0 \epsilon_r^2} R_H$$

Discrete states

$$E_n = E_g - \frac{1}{n^2} R_X$$

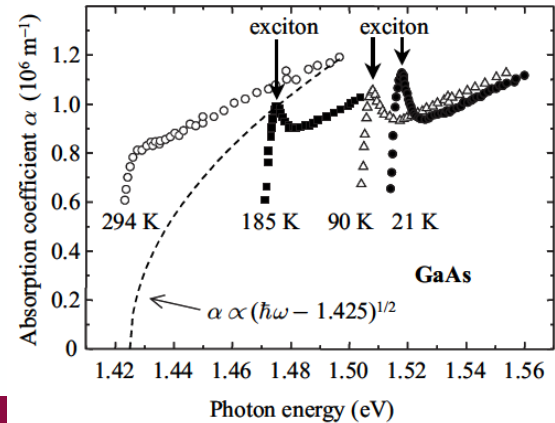
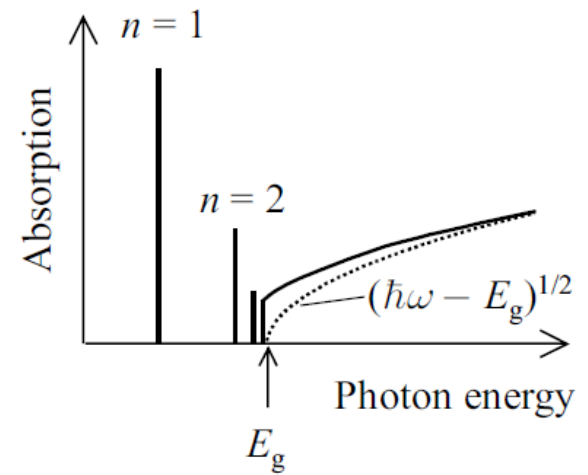
Discrete absorption

$$\epsilon_2(E) = \frac{8\pi |P|^2 \mu^3}{3\omega^2 (4\pi\epsilon_0)^3 \epsilon_r^3} \sum_{n=1}^{\infty} \frac{1}{n^3} \delta(E - E_n)$$

Continuum absorption

$$\epsilon_2(E) = \frac{2|P|^2 (2\mu)^{3/2} \sqrt{E - E_0}}{3\omega^2} \frac{\xi e^{\xi}}{\sinh \xi}$$

$$\xi = \pi \sqrt{R/E - E_0}$$



Use Bohr wave functions to calculate  $\epsilon_2$ .  
Toyozawa discusses broadening.

R. J. Elliott, Phys. Rev. **108**, 1384 (1957)  
Yu & Cardona; Fox, Chapter 4



# Elliott-Tanguy theory applied to Ge

## • Fixed parameters:

- Electron and hole masses (temperature dependent)
- Excitonic binding energy  $R$
- Amplitude  $A$  (derived from matrix element  $P$ )

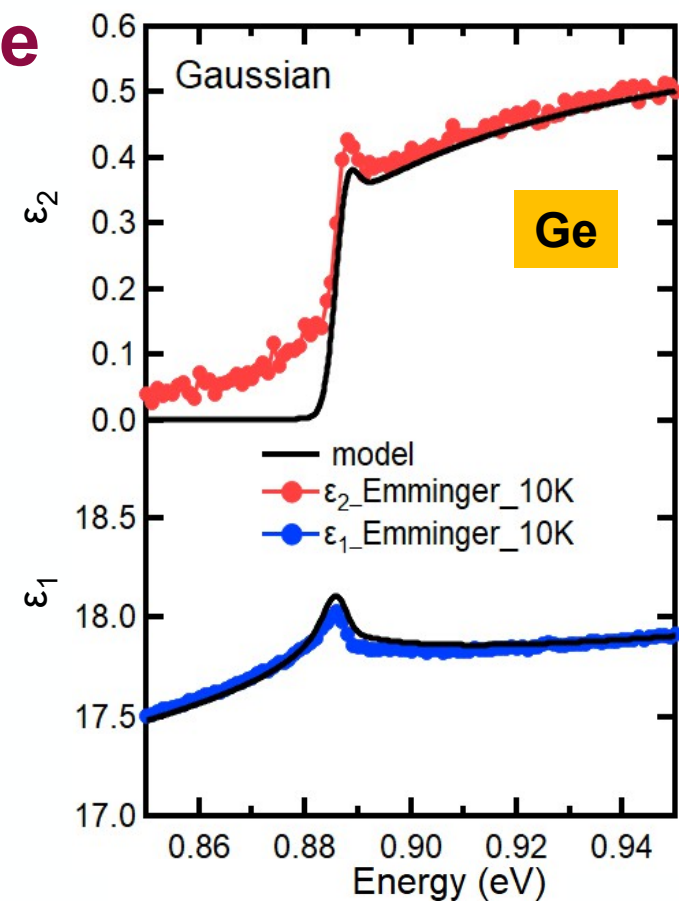
## • Adjustable parameters:

- Broadening  $\Gamma$ : 2.3 meV
- Band gap  $E_0$
- Linear background  $A_1$  and  $B_1$   
(contribution from  $E_1$  to real part of  $\epsilon$ )

Quantitative agreement

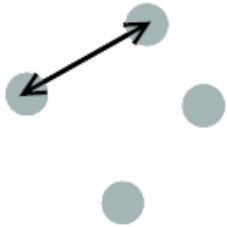
## • Problems:

- Broadening below the gap (band tail, oxide correction)



# Condensation of excitons at high density

## Exciton gas



(a) Low density  
Separation  $\gg$  diameter

Mott transition (insulator-metal) when electron separation equals exciton radius.

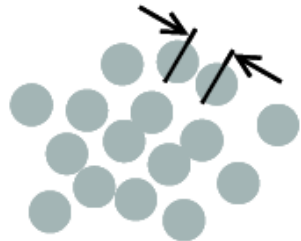
Electron separation  $d$  for density  $N$

$$d = \sqrt[3]{\frac{3}{4\pi n}}$$

$$r_s = \frac{d}{a_X}$$

dimensionless

## Electron-hole liquid



(b) High density  
Separation  $\approx$  diameter

Mott transition occurs at  $r_s$  near 1.

GaAs:  $n=10^{17} \text{ cm}^{-3}$ .

Biexciton, triexciton molecule formation.

Electron-hole droplets. Bose-Einstein condensation.

# Excitons in doped or excited semiconductors

Need to include exciton screening due to doping.

Yukawa potential: Schrödinger equation not solvable.

Use Hulthen potential as an approximation

Coulomb  $V(r) = -k \frac{1}{r}$

Yukawa  $V(r) = -k \frac{\exp(-r/\lambda_D)}{r}$

Hulthen  $V(r) = -k \frac{2/g a_X}{\exp\left(\frac{2r}{g a_X}\right) - 1}$

$$k = \frac{e^2}{4\pi\epsilon_0\epsilon_r}$$
$$\lambda_D = \sqrt{\frac{\epsilon_r\epsilon_0 k_B T}{ne^2}} = \frac{1}{k_D}$$

Debye screening length

$$g = \frac{\lambda_D}{a_X}$$

Unscreened:  $g=\infty$   
Fully screened:  $g=0$   
Mott criterion:  $g=1$

**Hulthen exciton** e Future.

C. Tanguy, Phys. Rev. **60**, 10660 (1999).  
Banyai & Koch, Z. Phys. B **63**, 283 (1986).

# Tanguy: Dielectric function of screened excitons

Bound exciton states (finite number):

$$A = \frac{\hbar^2 e^2}{6\pi\epsilon_0 m_0^2} \left(\frac{2\mu}{\hbar^2}\right)^{3/2} |P|^2$$

$$\epsilon_2(\omega) = \frac{2\pi A\sqrt{R}}{E^2} \sum_{n=1}^{n^2 < g} 2R \frac{1}{n} \left(\frac{1}{n^2} - \frac{n^2}{g^2}\right) \delta \left[ E - E_0 + \frac{R}{n^2} \left(1 - \frac{n^2}{g^2}\right)^2 \right]$$

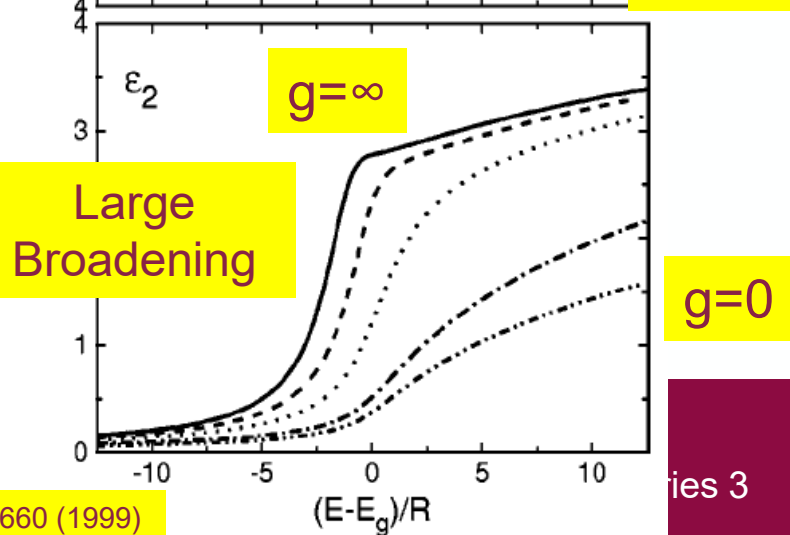
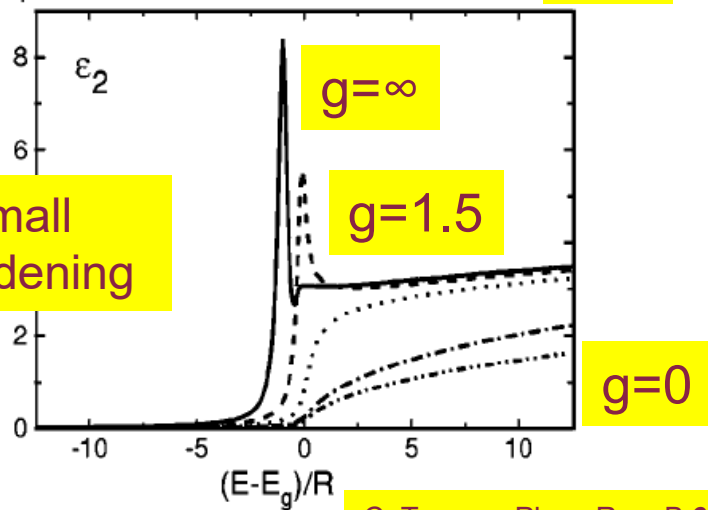
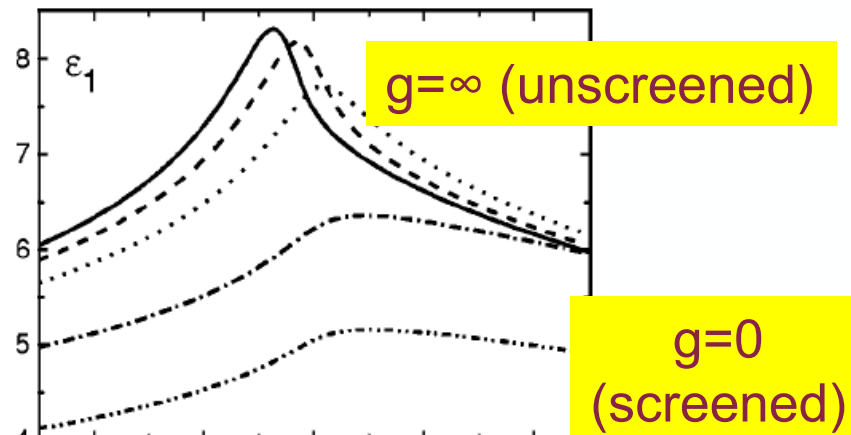
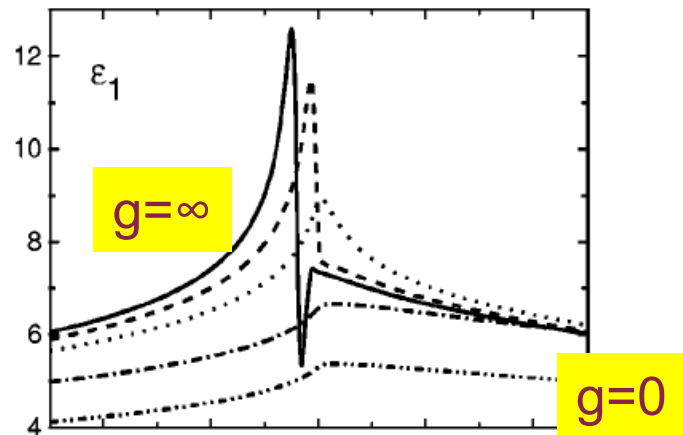
Reduced Rydberg energy

exciton continuum:

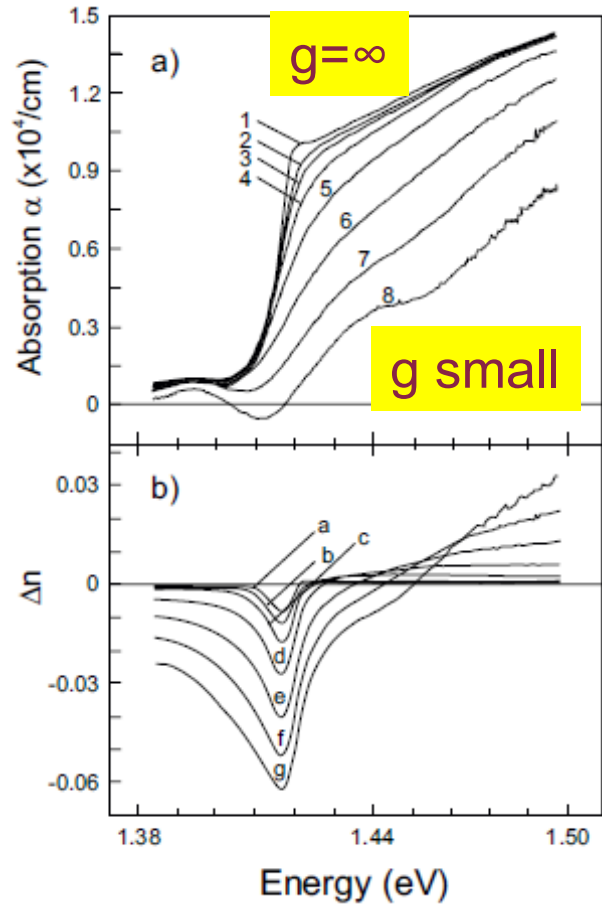
$$\epsilon_2(\omega) = \frac{2\pi A\sqrt{R}}{E^2} \frac{\sinh \pi g k}{\cosh(\pi g k) - \cosh\left(\pi g \sqrt{k^2 - \frac{4}{g}}\right)} \theta(E - E_0)$$
$$k = \pi \sqrt{(E - E_0)/R}$$

Need to introduce Lorentzian broadening and perform numerical KK transform.

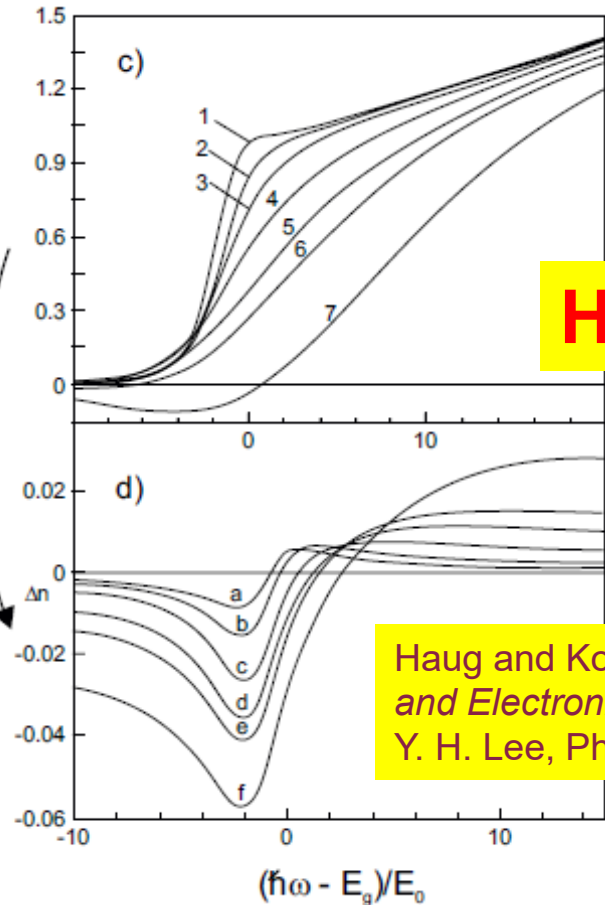
# Tanguy: Dielectric function of screened excitons



# Excitons in laser-excited GaAs



Kramers-Kronig



GaAs 300 K  
High laser excitation

**Hulthen exciton**

Haug and Koch, *Quantum Theory of Optical and Electronic Properties of Semiconductors*  
Y. H. Lee, Phys. Rev. Lett **57**, 2446 (1986)



# Two-dimensional Bohr problem

$$H = -\frac{\hbar^2}{2\mu_{\perp}} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) - \frac{\hbar^2}{2\mu_{\parallel}} \frac{\partial^2}{\partial z^2} - \frac{e^2}{\epsilon_r r}$$

Assume that  $\mu_{\parallel}$  is infinite (separate term).  
 Use cylindrical coordinates.  
 Separate radial and polar variables.  
 Similar Laguerre solution as 3D Bohr problem.

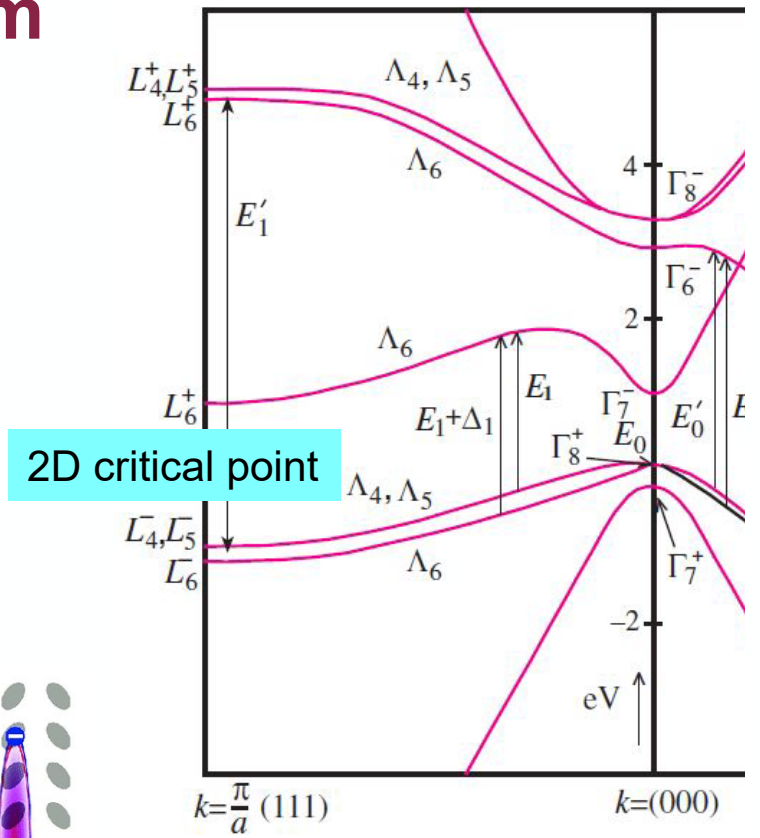
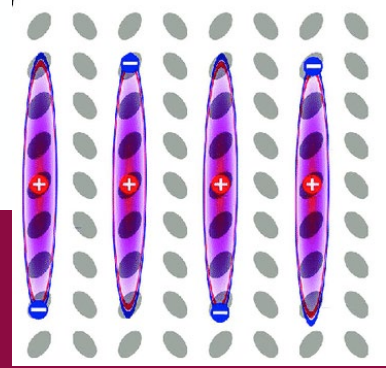
$$a_X = \frac{4\pi\epsilon_0\epsilon_r\hbar^2 m_0}{\mu_{\perp}\mu e^2}$$

$$R = \frac{\mu_{\perp} e^4}{2\hbar^2 m_0 (4\pi\epsilon_0\epsilon_r)^2}$$

$$E_n = -\frac{R}{\left(n - \frac{1}{2}\right)^2}, \quad n = 1, 2, \dots$$

Half-integral quantum numbers

STATE BE BOLD. Shape the Future.



M. Shinada and S. Sugano, J. Phys. Soc. Jpn. **21**, 1936 (1966).

# Two-dimensional saddle-point excitons ( $E_1, E_1 + \Delta_1$ )

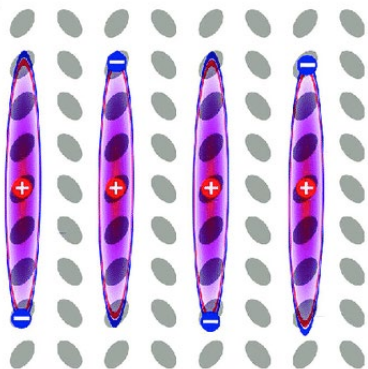
$$\varepsilon(E) = \frac{A}{(E + i\Gamma)^2} \{g[\xi(E + i\Gamma)] + g[\xi(-E - i\Gamma)] - 2g[\xi(0)]\}$$

$$g(\xi) = 2\ln(\xi) - 2\psi\left(\frac{1}{2} - \xi\right)$$

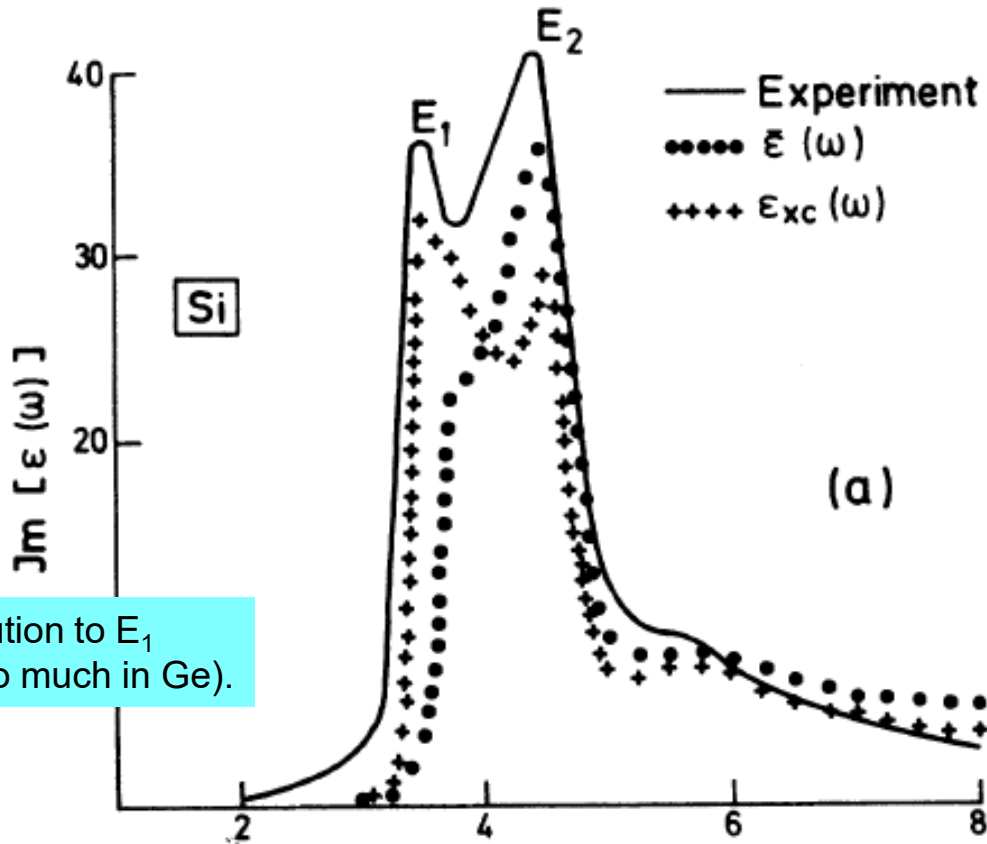
$$\psi(z) = \frac{d\ln\Gamma(z)}{dz}$$

$$\xi(z) = \sqrt{R/E_0 - z}$$

$$A = \frac{\mu e^2}{3\pi\varepsilon_0 m_0^2} |P|^2$$



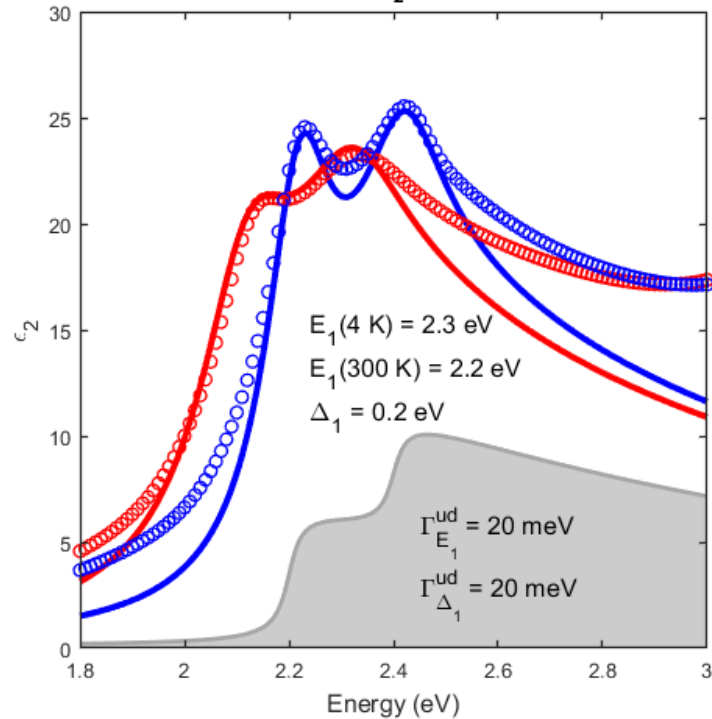
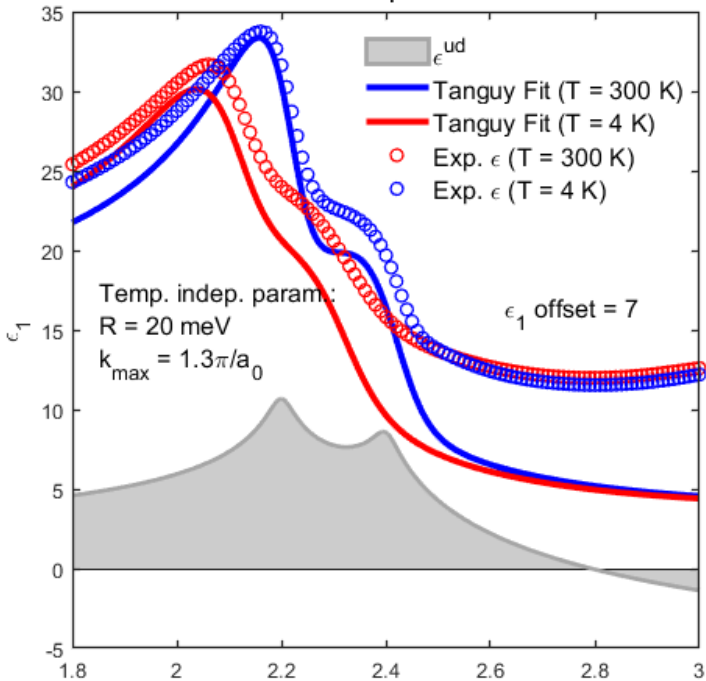
Strong excitonic contribution to  $E_1$  critical point in Si (not so much in Ge).



B. Velicky and J. Sak, *phys. status solidi* **16**, 147 (1966)  
 C. Tanguy, *Solid State Commun.* **98**, 65 (1996)  
 W. Hanke and L.J. Sham, *Phys. Rev. B* **21**, 4656 (1980)

# Comparison with experimental data

$$\varepsilon(E, E_1, \Gamma, R, k_{\max}) = \frac{k_{\max} e^2 \bar{P}^2 \mu_{\perp}^{(E_1)}}{3 \varepsilon_0 m^2 \pi (E + i\Gamma)^2} \left\{ g_a \left[ \sqrt{\frac{R}{E_1 - (E + i\Gamma)}} \right] + g_a \left[ \sqrt{\frac{R}{E_1 - (-E - i\Gamma)}} \right] - 2g_a \left[ \sqrt{\frac{R}{E_1 - (0)}} \right] \right\}$$



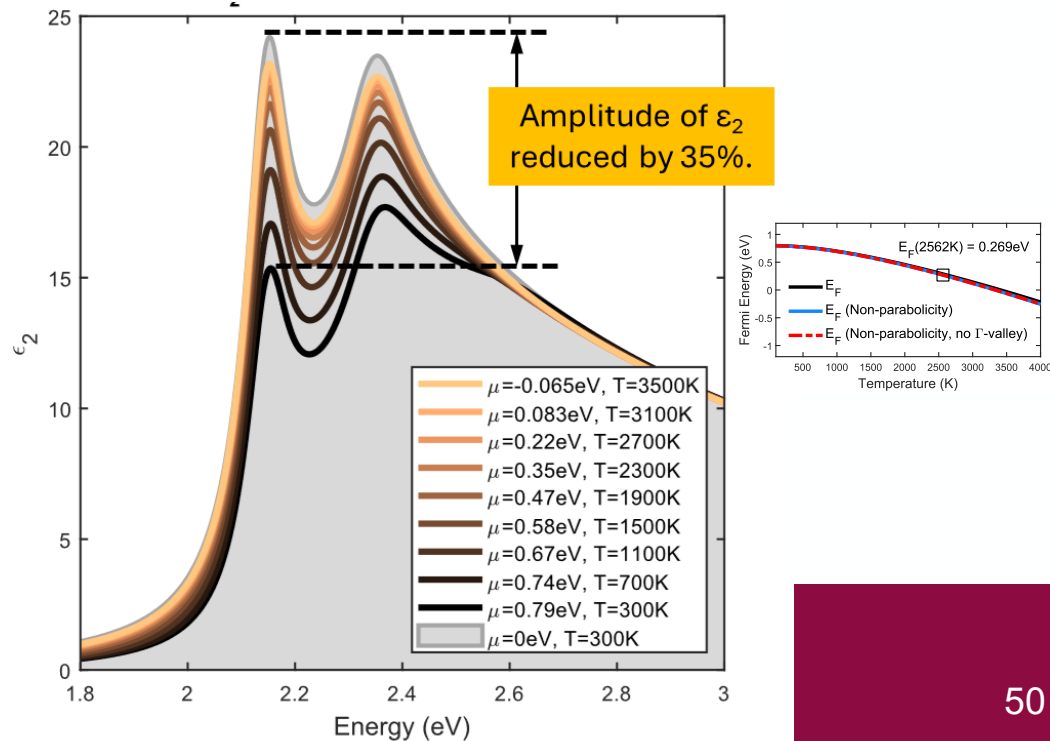
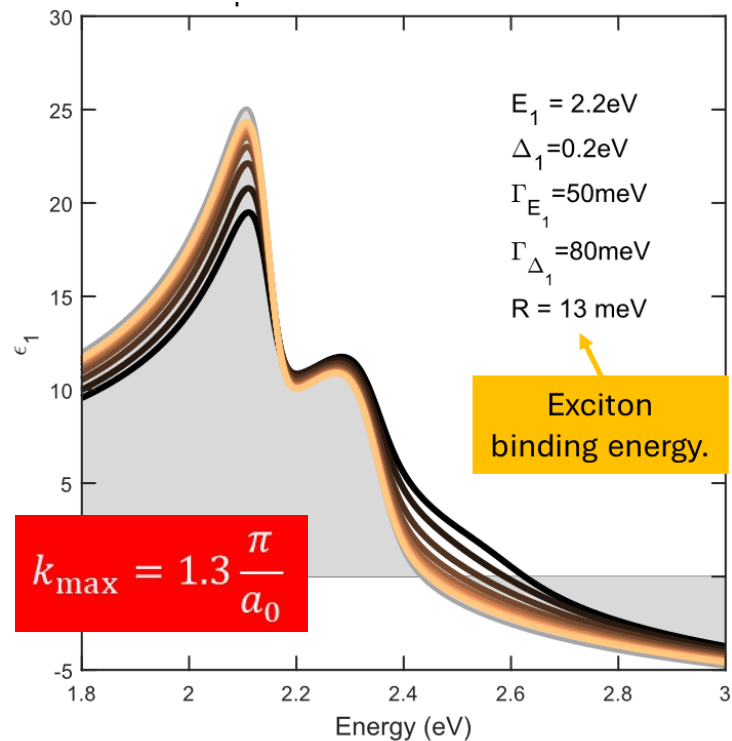
Experimental data:

Emminger (5 K),  
 JVST B **38**,  
 012202 (2020).

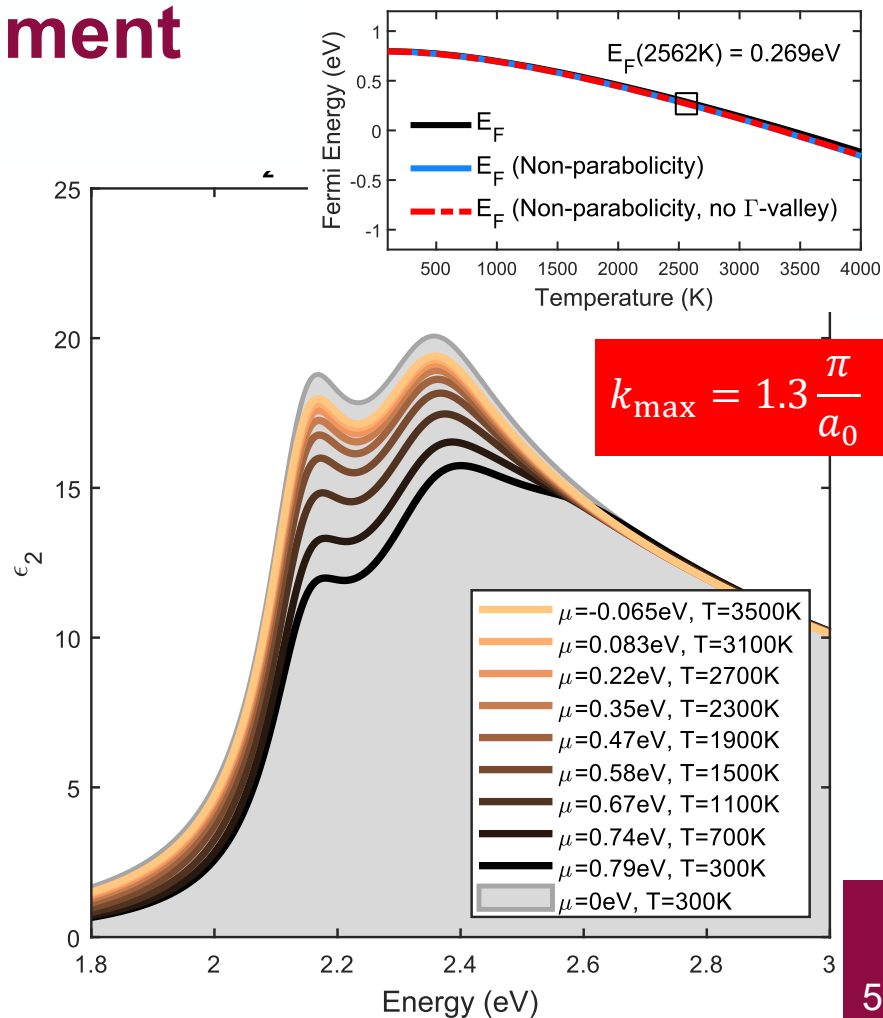
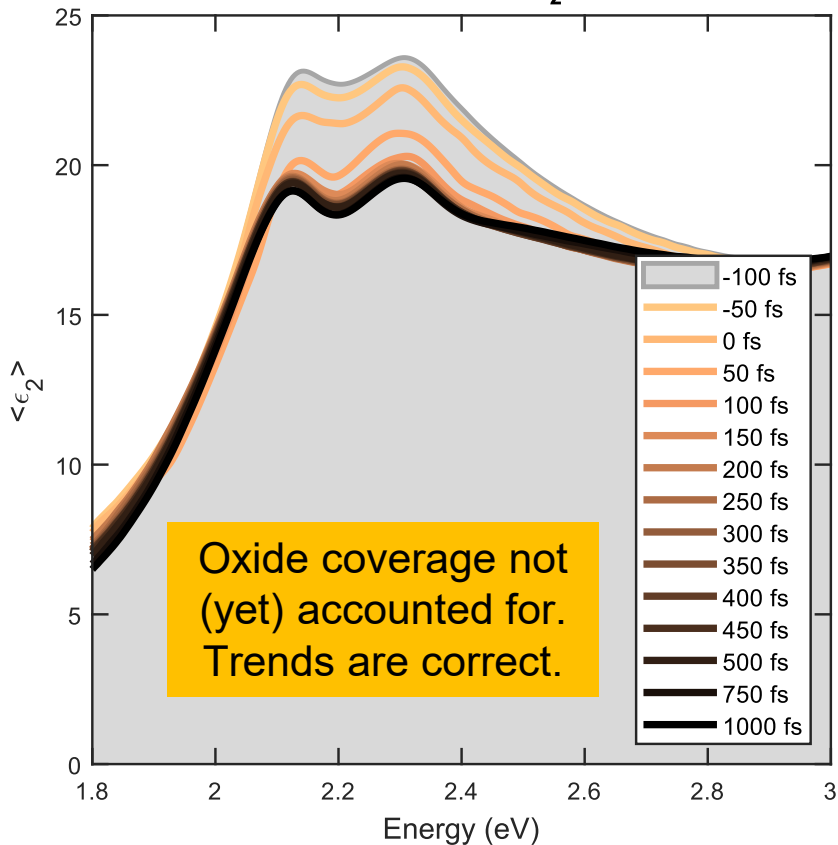
Nunley (300 K),  
 JVST B **34**,  
 061205 (2016)

# 2D excitons with band filling ( $E_1, E_1 + \Delta_1$ )

$$\epsilon_2(E) = \frac{e^2 \mu_{\perp}^{(E_1)} \bar{P}^2}{6\epsilon_0 m^2 \pi} \text{Im} \left\{ \frac{\{g_a[\xi(E + i\Gamma)] + g_a[\xi(-E - i\Gamma)] - 2g_a[\xi(0)]\}}{(E + i\Gamma)^2} \right\} \int_{-k_{\max}}^{k_{\max}} \{1 - f[E_c(E, k_z^2)]\} dk_z$$



# Comparison with experiment



# Conclusions

- Calculated electron-hole concentration (about  $10^{20} \text{ cm}^{-3}$ ) from Fermi-Dirac statistics.
- Electrons initially in  $\Gamma$ -valley, very hot electron plasma (2500 K)
- Within 50 fs, most electrons (>50%) scatter to the X-valley (large density of states).
- Electrons cool by intervalley scattering.  
When  $T < 1000 \text{ K}$ , most electrons are in the L-valley  
L-electrons are observable by bleaching the absorption of the probe pulse.
- Theory predicts a reduction of  $\varepsilon_2$  due to band filling (Pauli blocking) by about 20%.
- **A reduction by 25% is observed in the experiment, but the amplitude and line shape are wrong.**
- **Low-density dielectric function can be modeled by 2D-excitons.**
- **Band filling effects are in good agreement with transient dielectric function.**
- **What is missing?**
- **Excitonic (Sommerfeld) enhancement is screened by high electron density (TBD).**





**Thank you!**

**Questions?**

**Many students  
contributed to  
this project.**

<http://femto.nmsu.edu>